1. (5 points) Find the maximum flow from node A to node Y for the weighted directed graph below. Show the flow graph and the residual graph that result from this process.

![Graph Image]

2. (5 points) Use the depth-first search approach to determine the articulation points of the graph illustrated below. Show the depth-first tree for the graph, including the Num and Low values for each vertex, and specify the final list of articulation points.

![Graph Image]

3. (5 points) The pseudocode below performs a recursive depth-first search on a directed graph. (Note that n is assumed to be the number of vertices in the graph, and that each vertex is assumed to have an integer index value.)

```c
void depthFirstSearch(vertex v, bool visited[n])
{
    vertex w;
    visited[index(v)] = true;
    for each vertex w for which (v,w) is an edge
        if (!visited[w])
            depthFirstSearch(w, visited);
}
void DFS()
{
    bool visited[n] = {false};
    for each vertex v with (visited[index(v)] == false)
        depthFirstSearch(v, visited);
}
```

Specify the minor changes to this pseudocode that would cause it to output a topological sort of a directed acyclic graph in reverse order.
4. **EXTRA CREDIT (5 points)** Use the depth-first search approach to determine the strong components of the directed graph illustrated below. Show the depth-first forest for the original graph, the numbering of the reversed graph vertices, the depth-first forest for the reversed graph, and the final list of strong components.

![Graph Image]

5. **EXTRA CREDIT (5 points)** Use a depth-first search to find an Euler circuit for the graph below. Specify the individual cycles in the order of their removal and show how they are spliced together to form the Euler circuit.

![Graph Image]

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**You must provide your own solutions to these problems in a clearly presented Word document.**

**Obtaining solutions from any outside source is considered academic misconduct.**

**The only person with whom you may discuss these problems is the instructor.**