1. Answer each of the following questions, including a proof of your response.
   (a) (2 points) What is the largest number of nodes that can exist in a tree that is both a minimum heap and a binary search tree, assuming that duplicate values are not allowed?
   (b) (2 points) What is the largest number of nodes that can exist in a tree that is both a maximum heap and a binary search tree, assuming that duplicate values are not allowed?

2. Assume that you are trying to find the maximum element in an n-node minimum heap.
   (a) (2 points) Prove that it must be one of the leaf nodes.
   (b) (3 points) Use induction to prove that there are exactly ⌈n/2⌉ leaf nodes.
   (c) (2 points) Explain why it is necessary to examine every leaf node.

3. Show the result of inserting the numbers 1 through 15 into an initially empty leftist heap, assuming that the numbers are inserted:
   (a) (2 points) In order.
   (b) (2 points) In reverse order.

You must provide your own solutions to these problems in a clearly presented Word document. Obtaining solutions from any outside source is considered academic misconduct. The only person with whom you may discuss these problems is the instructor.