Chapter 9: Basic Cryptography

- Classical Cryptography
- Public Key Cryptography
- Cryptographic Checksums

Overview

- Classical Cryptography
  - Cæsar cipher
  - Vigènere cipher
  - DES
- Public Key Cryptography
  - Diffie-Hellman
  - RSA
- Cryptographic Checksums
  - HMAC

Cryptosystem

- Quintuple ($\mathcal{E}$, $\mathcal{D}$, $\mathcal{M}$, $\mathcal{K}$, $\mathcal{C}$)
  - $\mathcal{M}$ set of plaintexts
  - $\mathcal{K}$ set of keys
  - $\mathcal{C}$ set of ciphertexts
  - $\mathcal{E}$ set of encryption functions e: $\mathcal{M} \times \mathcal{K} \rightarrow \mathcal{C}$
  - $\mathcal{D}$ set of decryption functions d: $\mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$

Example

- Example: Cæsar cipher
  - $\mathcal{M}$ = { sequences of letters }
  - $\mathcal{K} = \{ i \mid i$ is an integer and $0 \le i \le 25 \}$
  - $\mathcal{E} = \{ E_k \mid k \in \mathcal{K}$ and for all letters m,

    $E_k(m) = (m + k) \bmod 26 \}$
  - $\mathcal{D} = \{ D_k \mid k \in \mathcal{K}$ and for all letters c,

    $D_k(c) = (26 + c - k) \bmod 26 \}$
  - $\mathcal{C} = \mathcal{M}$

Attacks

- Opponent whose goal is to break cryptosystem is the *adversary*
  - Assume adversary knows algorithm used, but not key
- Three types of attacks:
  - *ciphertext only*: adversary has only ciphertext; goal is to find plaintext, possibly key
  - *known plaintext*: adversary has ciphertext, corresponding plaintext; goal is to find key
  - *chosen plaintext*: adversary may supply plaintexts and obtain corresponding ciphertext; goal is to find key

Basis for Attacks

- Mathematical attacks
  - Based on analysis of underlying mathematics
- Statistical attacks
  - Make assumptions about the distribution of letters, pairs of letters (digrams), triplets of letters (trigrams), *etc*.
    - Called *models of the language*
  - Examine ciphertext, correlate properties with the assumptions.

Classical Cryptography

- Sender, receiver share common key
  - Keys may be the same, or trivial to derive from one another
  - Sometimes called *symmetric cryptography*
- Two basic types
  - Transposition ciphers
  - Substitution ciphers
  - Combinations are called *product ciphers*

Transposition Cipher

- Rearrange letters in plaintext to produce ciphertext
- Example (Rail-Fence Cipher)
  - Plaintext is `HELLO WORLD`
  - Rearrange as

```
HLOOL
ELWRD
```

  - Ciphertext is `HLOOL ELWRD`

Attacking the Cipher

- Anagramming
  - If 1-gram frequencies match English frequencies, but other *n*-gram frequencies do not, probably transposition
  - Rearrange letters to form *n*-grams with highest frequencies

Example

- Ciphertext: `HLOOLELWRD`
- Frequencies of 2-grams beginning with H
  - HE 0.0305
  - HO 0.0043
  - HL, HW, HR, HD < 0.0010
- Frequencies of 2-grams ending in H
  - WH 0.0026
  - EH, LH, OH, RH, DH ≤ 0.0002
- Implies E follows H

Example

- Arrange so the H and E are adjacent

```
HE
LL
OW
OR
LD
```

- Read off across, then down, to get original plaintext

Substitution Ciphers

- Change characters in plaintext to produce ciphertext
- Example (Cæsar cipher)
  - Plaintext is `HELLO WORLD`
  - Change each letter to the third letter following it (X goes to A, Y to B, Z to C)
    - Key is 3, usually written as letter 'D'
  - Ciphertext is `KHOOR ZRUOG`

Attacking the Cipher

- Exhaustive search
  - If the key space is small enough, try all possible keys until you find the right one
  - Cæsar cipher has 26 possible keys
- Statistical analysis
  - Compare to 1-gram model of English

Statistical Attack

- Compute frequency of each letter in ciphertext:

G	0.1	H	0.1	K	0.1	O	0.3
R	0.2	U	0.1	Z	0.1		

- Apply 1-gram model of English
  - Frequency of characters (1-grams) in English is on next slide

Character Frequencies

a	0.080	h	0.060	n	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c	0.030	j	0.005	p	0.020	v	0.010
d	0.040	k	0.005	q	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g	0.015					z	0.002

Statistical Analysis

- $f(c)$ frequency of character c in ciphertext
- $\varphi(i)$ correlation of frequency of letters in ciphertext with corresponding letters in English, assuming key is i
  - $\varphi(i) = \Sigma_{0 \le c \le 25} f(c)p(c - i)$ so here,
    $\varphi(i) = 0.1p(6 - i) + 0.1p(7 - i) + 0.1p(10 - i) + 0.3p(14 - i) + 0.2p(17 - i) + 0.1p(20 - i) + 0.1p(25 - i)$
    - $p(x)$ is frequency of character x in English

Correlation: $\varphi(i)$ for $0 \leq i \leq 25$

i	**φ(*i*)**	***i***	**φ(*i*)**	***i***	**φ(*i*)**	***i***	**φ(*i*)**
0	0.0482	7	0.0442	13	0.0520	19	0.0315
1	0.0364	8	0.0202	14	0.0535	20	0.0302
2	0.0410	9	0.0267	15	0.0226	21	0.0517
3	0.0575	10	0.0635	16	0.0322	22	0.0380
4	0.0252	11	0.0262	17	0.0392	23	0.0370
5	0.0190	12	0.0325	18	0.0299	24	0.0316
6	0.0660					25	0.0430

The Result

- Most probable keys, based on φ:
  - $i = 6$, $\varphi(i) = 0.0660$
    - plaintext `EBIIL TLOLA`
  - $i = 10$, $\varphi(i) = 0.0635$
    - plaintext `AXEEH PHKEW`
  - $i = 3$, $\varphi(i) = 0.0575$
    - plaintext `HELLO WORLD`
  - $i = 14$, $\varphi(i) = 0.0535$
    - plaintext `WTAAD LDGAS`
- Only English phrase is for $i = 3$
  - That's the key (3 or 'D')

Cæsar's Problem

- Key is too short
  - Can be found by exhaustive search
  - Statistical frequencies not concealed well
    - They look too much like regular English letters
- So make it longer
  - Multiple letters in key
  - Idea is to smooth the statistical frequencies to make cryptanalysis harder

Vigènere Cipher

- Like Cæsar cipher, but use a phrase
- Example
  - Message `THE BOY HAS THE BALL`
  - Key `VIG`
  - Encipher using Cæsar cipher for each letter:

```
key    VIGVIGVIGVIGVIGV
plain  THEBOYHASTHEBALL
cipher OPKWWECIYOPKWIRG
```

Relevant Parts of Tableau

	G	*I*	*V*
A	G	I	V
B	H	J	W
E	L	M	Z
H	N	P	C
L	R	T	G
O	U	W	J
S	Y	A	N
T	Z	B	O
Y	E	H	T

- Tableau shown has relevant rows, columns only
- Example encipherments:
  - key V, letter T: follow V column down to T row (giving "O")
  - Key I, letter H: follow I column down to H row (giving "P")

Useful Terms

- *period*: length of key
  - In earlier example, period is 3
- *tableau*: table used to encipher and decipher
  - Vigènere cipher has key letters on top, plaintext letters on the left
- *polyalphabetic*: the key has several different letters
  - Cæsar cipher is monoalphabetic

Attacking the Cipher

- Approach
  - Establish period; call it *n*
  - Break message into *n* parts, each part being enciphered using the same key letter
  - Solve each part
    - You can leverage one part from another
- We will show each step

The Target Cipher

- We want to break this cipher:

```
ADQYS MIUSB OXKKT MIBHK IZOOO
EQOOG IFBAG KAUMF VVTAA CIDTW
MOCIO EQOOG BMBFV ZGGWP CIEKQ
HSNEW VECNE DLAAV RWKXS VNSVP
HCEUT QOIOF MEGJS WTPCH AJMOC
HIUIX
```

Establish Period

- Kaskski: *repetitions in the ciphertext occur when characters of the key appear over the same characters in the plaintext*
- Example:

```
key    VIGVIGVIGVIGVIGV
plain  THEBOYHASTHEBALL
cipher OPKWWECIYOPKWIRG
```

Note the key and plaintext line up over the repetitions (underlined). As distance between repetitions is 9, the period is a factor of 9 (that is, 1, 3, or 9)

Repetitions in Example

Letters	*Start*	*End*	*Distance*	*Factors*
MI	5	15	10	2, 5
OO	22	27	5	5
OEQOOG	24	54	30	2, 3, 5
FV	39	63	24	2, 2, 2, 3
AA	43	87	44	2, 2, 11
MOC	50	122	72	2, 2, 2, 3, 3
QO	56	105	49	7, 7
PC	69	117	48	2, 2, 2, 2, 3
NE	77	83	6	2, 3
SV	94	97	3	3
CH	118	124	6	2, 3

Estimate of Period

- OEQOOG is probably not a coincidence
  - It's too long for that
  - Period may be 1, 2, 3, 5, 6, 10, 15, or 30
- Most others (7/10) have 2 in their factors
- Almost as many (6/10) have 3 in their factors
- Begin with period of $2 \times 3 = 6$

Check on Period

- Index of coincidence is probability that two randomly chosen letters from ciphertext will be the same
- Tabulated for different periods:

1	0.066	3	0.047	5	0.044
2	0.052	4	0.045	10	0.041
Large	0.038				

Compute IC

- IC = $[n\,(n-1)]^{-1} \sum_{0 \le i \le 25} [F_i\,(F_i - 1)]$
  - where n is length of ciphertext and F_i the number of times character i occurs in ciphertext
- Here, IC = 0.043
  - Indicates a key of slightly more than 5
  - A statistical measure, so it can be in error, but it agrees with the previous estimate (which was 6)

Splitting Into Alphabets

alphabet 1: `AIKHOIATTOBGEEERNEOSAI`

alphabet 2: `DUKKEFUAWEMGKWDWSUFWJU`

alphabet 3: `QSTIQBMAMQBWQVLKVTMTMI`

alphabet 4: `YBMZOAFCOOFPHEAXPQEPOX`

alphabet 5: `SOIOOGVICOVCSVASHOGCC`

alphabet 6: `MXBOGKVDIGZINNVVCIJHH`

- ICs (#1, 0.069; #2, 0.078; #3, 0.078; #4, 0.056; #5, 0.124; #6, 0.043) indicate all alphabets have period 1, except #4 and #6; assume statistics off

Frequency Examination

```
    ABCDEFGHIJKLMNOPQRSTUVWXYZ
1   31004011301001300112000000
2   10022210013010000010404000
3   12000000201140004013021000
4   21102201000001043100000211
5   10500021200000500030020000
6   01110022311012100000030101
```

Letter frequencies are (H high, M medium, L low):

```
    HMMMHMMHHMMMMHHMLHHHMLLLLL
```

Begin Decryption

- First matches characteristics of unshifted alphabet
- Third matches if I shifted to A
- Sixth matches if V shifted to A
- Substitute into ciphertext (bold are substitutions)

AD**I**YS **RI**U**K**B O**CK**K**L** MI**GH**K **A**ZO**TO**
E**I**OO**L** **I**F**T**AG **PA**U**E**F V**AT**A**S** CI**IT**W
EOC**NO** E**I**OO**L** **B**M**T**FV **EG**G**O**P C**NE**K**I**
HS**SE**W **N**EC**SE** D**D**AA**A** **R**W**C**XS **AN**S**N**P
H**HE**U**L** QO**NO**F **E**EG**OS** W**L**PC**M** **AJE**OC
MIU**A**X

Look For Clues

- **AJE** in last line suggests "are", meaning second alphabet maps A into S:

ALIYS **RICK**B O**CKSL** MI**GHS** **A**ZO**TO**

MIOO**L** **INT**AG **PACE**F V**ATIS** CI**ITE**

EOC**NO** **MI**OO**L** **BUT**FV **EGOO**P C**NESI**

HS**SEE** **N**EC**SE** **LD**AA**A** **REC**XS **ANAN**P

H**HECL** QO**NON** **E**EG**OS** **EL**PCM **ARE**OC

MICAX

Next Alphabet

- **MICA**X in last line suggests "mical" (a common ending for an adjective), meaning fourth alphabet maps O into A:

ALIMS **RICKP** O**CKSL** **A**I**GHS** **AN**O**TO**
MICO**L** **INTO**G **PACET** V**ATIS** **Q**I**ITE**
ECC**NO** **MIC**O**L** **BUTT**V **EGOOD** C**NESI**
VS**SEE** **NS**C**SE** **LDO**A**A** **RECL**S **ANAND**
H**HECL** **E**O**NON** **ES**G**OS** **ELD**CM **AREC**C
MICAL

Got It!

- QI means that U maps into I, as Q is always followed by U:

ALIME RICKP ACKSL AUGHS ANATO
MICAL INTOS PACET HATIS QUITE
ECONO MICAL BUTTH EGOOD ONESI
VESEE NSOSE LDOMA RECLE ANAND
THECL EANON ESSOS ELDOM ARECO
MICAL

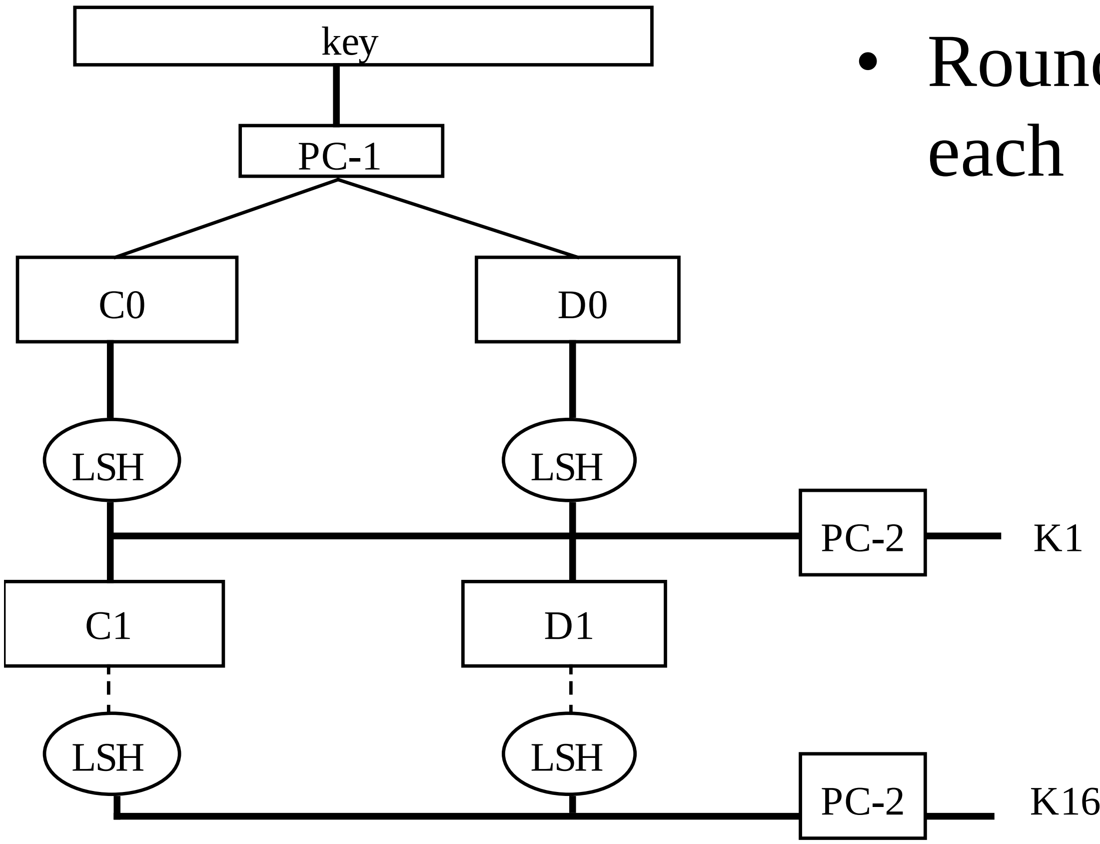
One-Time Pad

- A Vigenère cipher with a random key at least as long as the message
  - Provably unbreakable
  - Why? Look at ciphertext `DXQR`. Equally likely to correspond to plaintext `DOIT` (key `AJIY`) and to plaintext `DONT` (key `AJDY`) and any other 4 letters
  - Warning: keys *must* be random, or you can attack the cipher by trying to regenerate the key
    - Approximations, such as using pseudorandom number generators to generate keys, are *not* random

Overview of the DES

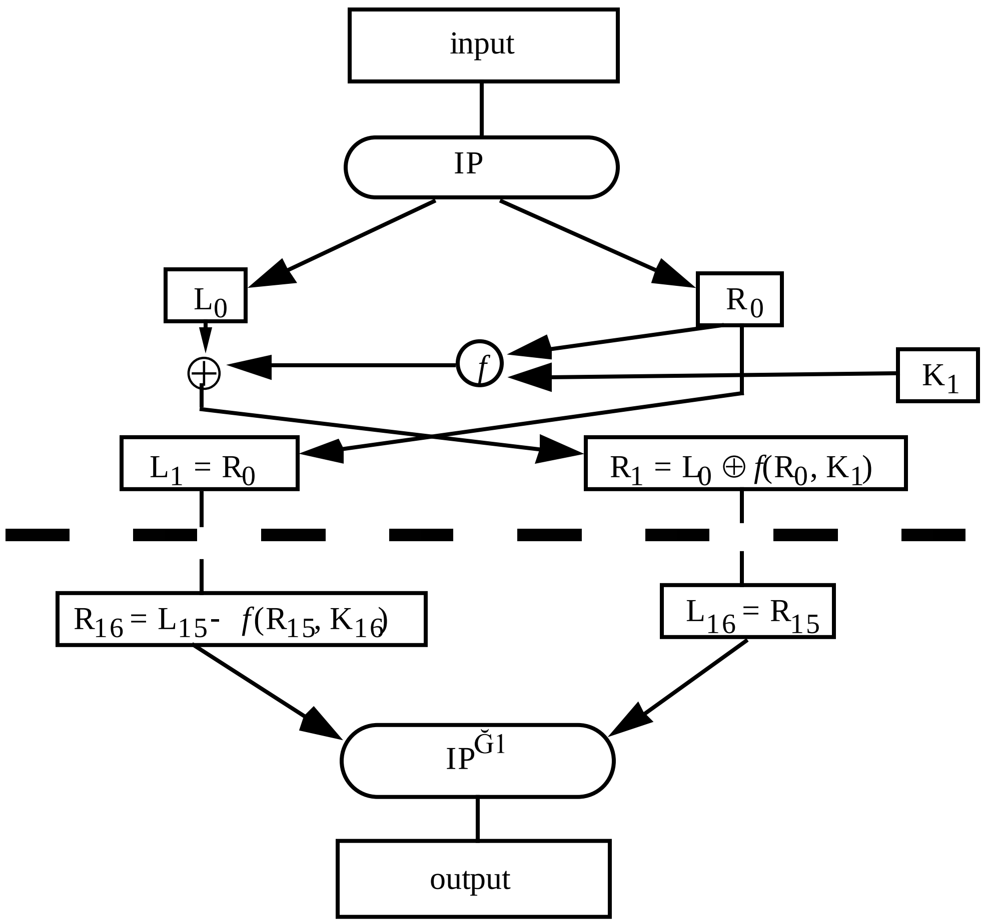
- A block cipher:
  - encrypts blocks of 64 bits using a 64 bit key
  - outputs 64 bits of ciphertext
- A product cipher
  - basic unit is the bit
  - performs both substitution and transposition (permutation) on the bits
- Cipher consists of 16 rounds (iterations) each with a round key generated from the user-supplied key

Generation of Round Keys

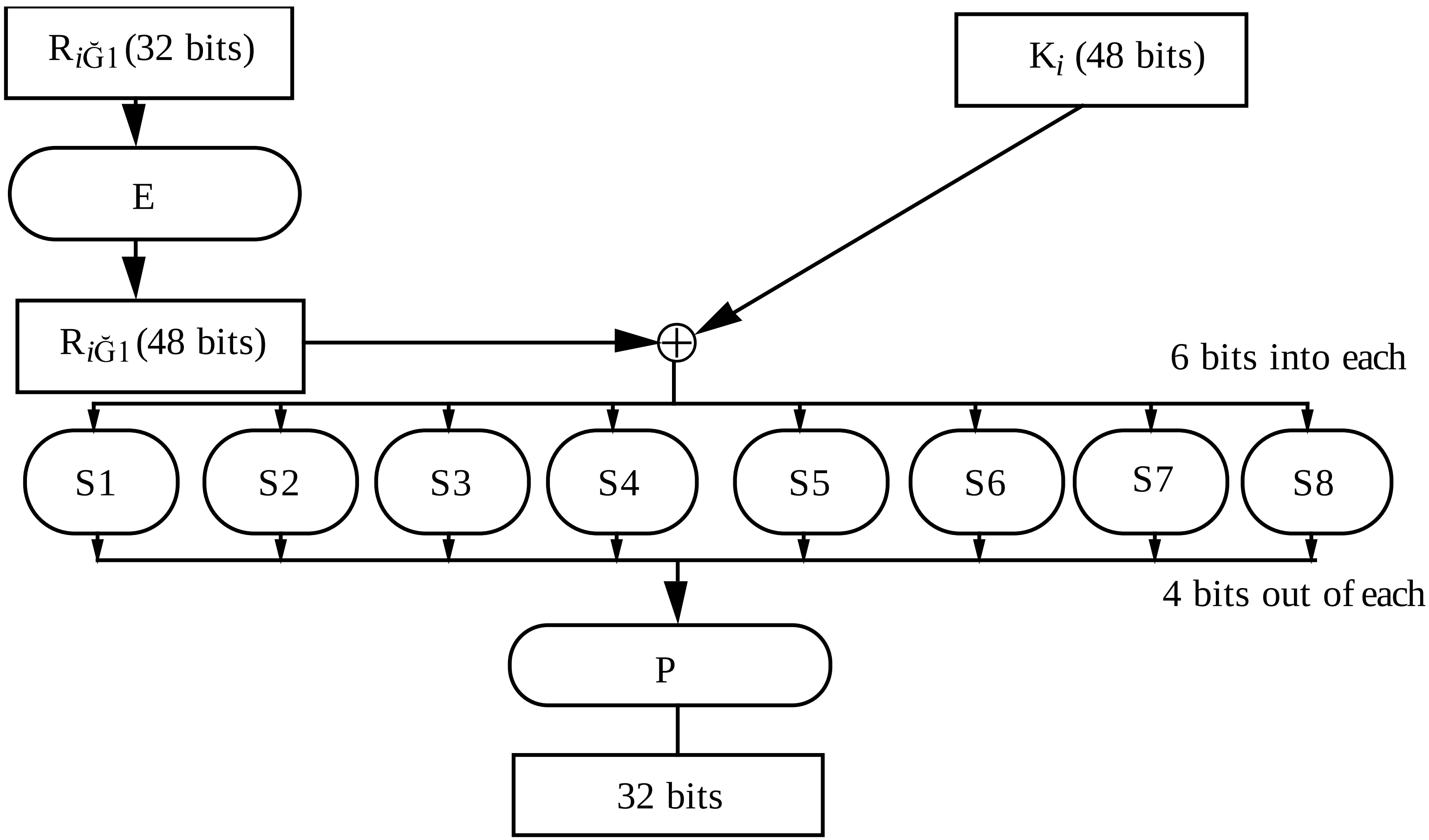


- Round keys are 48 bits each

Encipherment



The *f* Function



Controversy

- Considered too weak
  - Diffie, Hellman said in a few years technology would allow DES to be broken in days
    - Design using 1999 technology published
  - Design decisions not public
    - S-boxes may have backdoors

Undesirable Properties

- 4 weak keys
  - They are their own inverses
- 12 semi-weak keys
  - Each has another semi-weak key as inverse
- Complementation property
  - $\mathrm{DES}_k(m) = c \Rightarrow \mathrm{DES}_{k'}(m') = c'$
- S-boxes exhibit irregular properties
  - Distribution of odd, even numbers non-random
  - Outputs of fourth box depends on input to third box

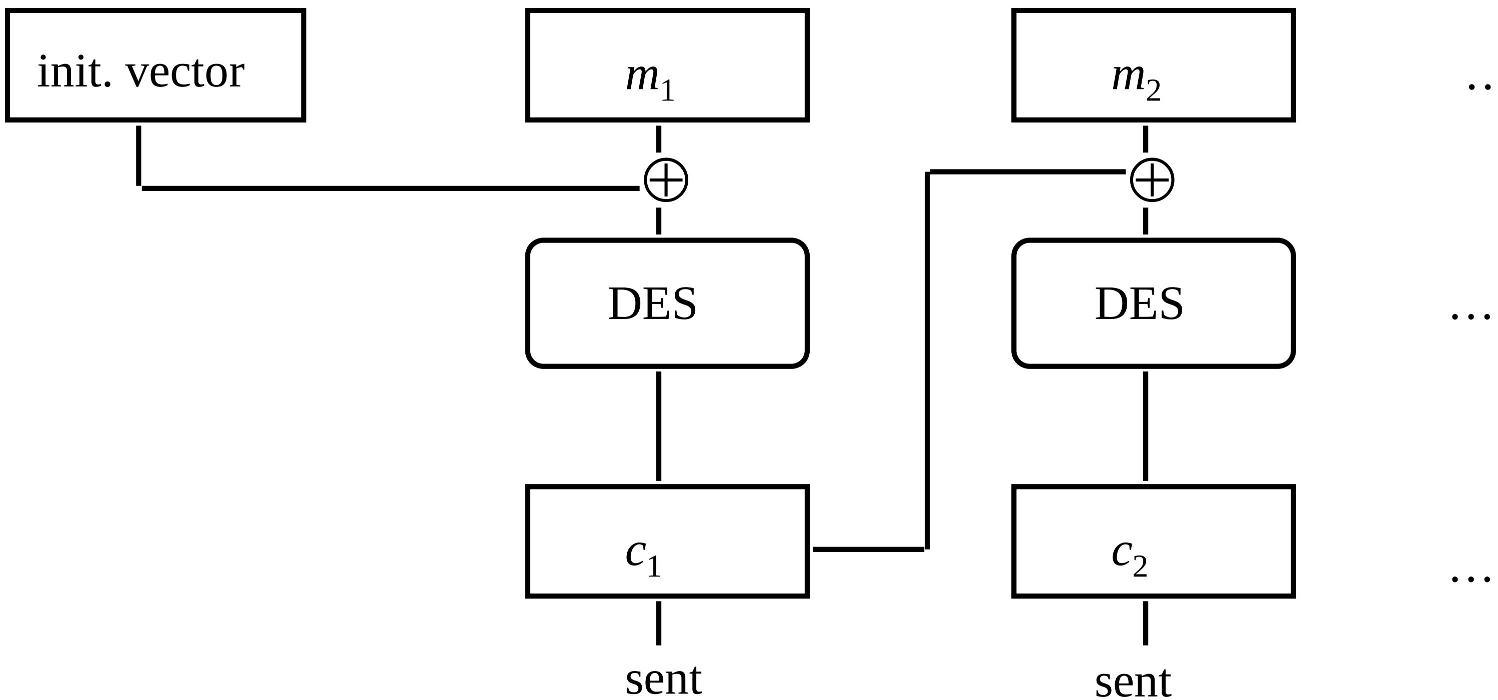
Differential Cryptanalysis

- A chosen ciphertext attack
  - Requires 2^{47} plaintext, ciphertext pairs
- Revealed several properties
  - Small changes in S-boxes reduce the number of pairs needed
  - Making every bit of the round keys independent does not impede attack
- Linear cryptanalysis improves result
  - Requires 2^{43} plaintext, ciphertext pairs

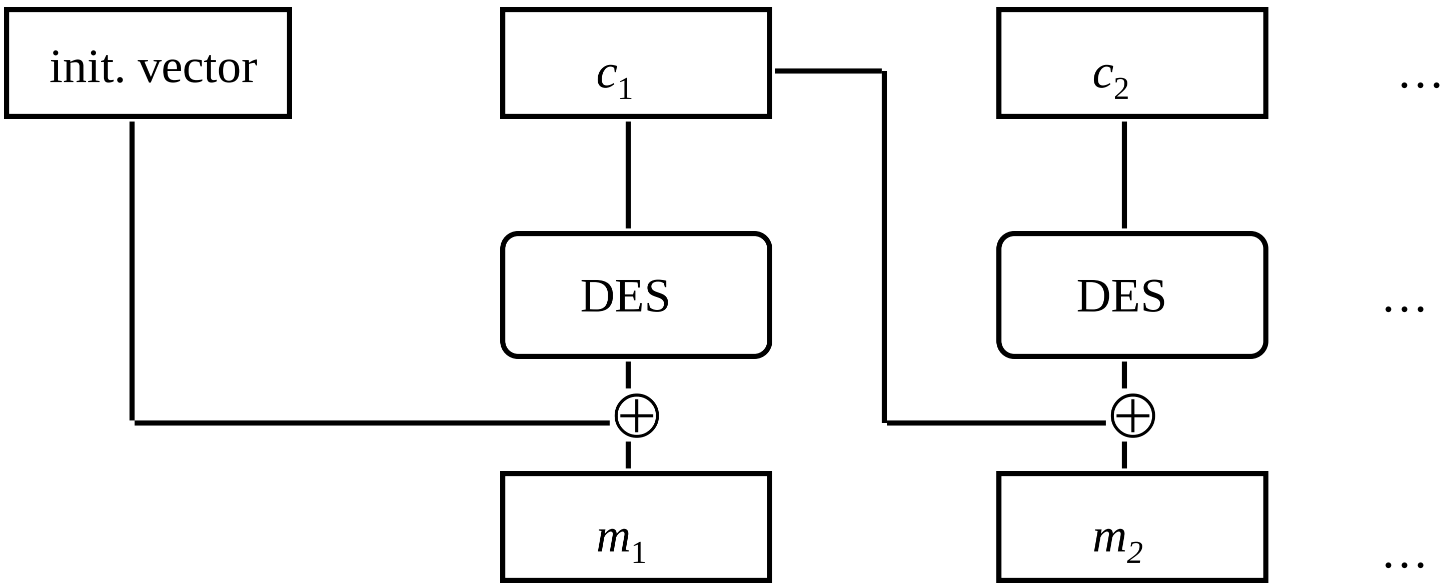
DES Modes

- Electronic Code Book Mode (ECB)
  - Encipher each block independently
- Cipher Block Chaining Mode (CBC)
  - Xor each block with previous ciphertext block
  - Requires an initialization vector for the first one
- Encrypt-Decrypt-Encrypt Mode (2 keys: k, k')
  - $c = \mathrm{DES}_k(\mathrm{DES}_{k'}^{-1}(\mathrm{DES}_k(m)))$
- Encrypt-Encrypt-Encrypt Mode (3 keys: k, k', k'')
  - $c = \mathrm{DES}_k(\mathrm{DES}_{k'}(\mathrm{DES}_{k''}(m)))$

CBC Mode Encryption



CBC Mode Decryption



Self-Healing Property

- Initial message
  - 3231343336353837 3231343336353837 3231343336353837 3231343336353837
- Received as (underlined 4c should be 4b)
  - ef7c<u>4c</u>b2b4ce6f3b f6266e3a97af0e2c 746ab9a6308f4256 33e60b451b09603d
- Which decrypts to
  - <u>efca61e19f4836f1</u> 3231<u>33</u>3336353837 3231343336353837 3231343336353837
  - Incorrect bytes underlined
  - Plaintext "heals" after 2 blocks

Current Status of DES

- Design for computer system, associated software that could break any DES-enciphered message in a few days published in 1998
- Several challenges to break DES messages solved using distributed computing
- NIST selected Rijndael as Advanced Encryption Standard, successor to DES
  - Designed to withstand attacks that were successful on DES

Public Key Cryptography

- Two keys
  - *Private key* known only to individual
  - *Public key* available to anyone
    - Public key, private key inverses
- Idea
  - Confidentiality: encipher using public key, decipher using private key
  - Integrity/authentication: encipher using private key, decipher using public one

Requirements

1. It must be computationally easy to encipher or decipher a message given the appropriate key
2. It must be computationally infeasible to derive the private key from the public key
3. It must be computationally infeasible to determine the private key from a chosen plaintext attack

Diffie-Hellman

- Compute a common, shared key
  - Called a *symmetric key exchange protocol*
- Based on discrete logarithm problem
  - Given integers n and g and prime number p, compute k such that $n = g^k \bmod p$
  - Solutions known for small p
  - Solutions computationally infeasible as p grows large

Algorithm

- Constants: prime p, integer $g \neq 0, 1, p-1$
  - Known to all participants
- Anne chooses private key *kAnne*, computes public key $KAnne = g^{kAnne} \bmod p$
- To communicate with Bob, Anne computes $Kshared = KBob^{kAnne} \bmod p$
- To communicate with Anne, Bob computes $Kshared = KAnne^{kBob} \bmod p$
  - It can be shown these keys are equal

Example

- Assume p = 53 and g = 17
- Alice chooses *kAlice* = 5
  - Then *KAlice* = 17^5 mod 53 = 40
- Bob chooses *kBob* = 7
  - Then *KBob* = 17^7 mod 53 = 6
- Shared key:
  - $KBob^{kAlice}$ mod p = 6^5 mod 53 = 38
  - $KAlice^{kBob}$ mod p = 40^7 mod 53 = 38

RSA

- Exponentiation cipher
- Relies on the difficulty of determining the number of numbers relatively prime to a large integer n

Background

- Totient function $\phi(n)$
  – Number of positive integers less than *n* and relatively prime to *n*
    - *Relatively prime* means with no factors in common with *n*
- Example: $\phi(10) = 4$
  – 1, 3, 7, 9 are relatively prime to 10
- Example: $\phi(21) = 12$
  – 1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20 are relatively prime to 21

Algorithm

- Choose two large prime numbers p, q
  - Let $n = pq$; then $\phi(n) = (p-1)(q-1)$
  - Choose $e < n$ such that e is relatively prime to $\phi(n)$.
  - Compute d such that $ed \bmod \phi(n) = 1$
- Public key: (e, n); private key: d
- Encipher: $c = m^e \bmod n$
- Decipher: $m = c^d \bmod n$

Example: Confidentiality

- Take $p = 7$, $q = 11$, so $n = 77$ and $\phi(n) = 60$
- Alice chooses $e = 17$, making $d = 53$
- Bob wants to send Alice secret message HELLO (07 04 11 11 14)
  - $07^{17} \bmod 77 = 28$
  - $04^{17} \bmod 77 = 16$
  - $11^{17} \bmod 77 = 44$
  - $11^{17} \bmod 77 = 44$
  - $14^{17} \bmod 77 = 42$
- Bob sends 28 16 44 44 42

Example

- Alice receives 28 16 44 44 42
- Alice uses private key, d = 53, to decrypt message:
  - $28^{53} \bmod 77 = 07$
  - $16^{53} \bmod 77 = 04$
  - $44^{53} \bmod 77 = 11$
  - $44^{53} \bmod 77 = 11$
  - $42^{53} \bmod 77 = 14$
- Alice translates message to letters to read HELLO
  - No one else could read it, as only Alice knows her private key and that is needed for decryption

Example: Integrity/Authentication

- Take $p = 7$, $q = 11$, so $n = 77$ and $\phi(n) = 60$
- Alice chooses $e = 17$, making $d = 53$
- Alice wants to send Bob message HELLO (07 04 11 11 14) so Bob knows it is what Alice sent (no changes in transit, and authenticated)
  - $07^{53} \bmod 77 = 35$
  - $04^{53} \bmod 77 = 09$
  - $11^{53} \bmod 77 = 44$
  - $11^{53} \bmod 77 = 44$
  - $14^{53} \bmod 77 = 49$
- Alice sends 35 09 44 44 49

Example

- Bob receives 35 09 44 44 49
- Bob uses Alice's public key, $e = 17$, $n = 77$, to decrypt message:
  - $35^{17} \bmod 77 = 07$
  - $09^{17} \bmod 77 = 04$
  - $44^{17} \bmod 77 = 11$
  - $44^{17} \bmod 77 = 11$
  - $49^{17} \bmod 77 = 14$
- Bob translates message to letters to read HELLO
  - Alice sent it as only she knows her private key, so no one else could have enciphered it
  - If (enciphered) message's blocks (letters) altered in transit, would not decrypt properly

Example: Both

- Alice wants to send Bob message HELLO both enciphered and authenticated (integrity-checked)
  - Alice's keys: public (17, 77); private: 53
  - Bob's keys: public: (37, 77); private: 13
- Alice enciphers HELLO (07 04 11 11 14):
  - $(07^{53} \bmod 77)^{37} \bmod 77 = 07$
  - $(04^{53} \bmod 77)^{37} \bmod 77 = 37$
  - $(11^{53} \bmod 77)^{37} \bmod 77 = 44$
  - $(11^{53} \bmod 77)^{37} \bmod 77 = 44$
  - $(14^{53} \bmod 77)^{37} \bmod 77 = 14$
- Alice sends 07 37 44 44 14

Security Services

- Confidentiality
  - Only the owner of the private key knows it, so text enciphered with public key cannot be read by anyone except the owner of the private key
- Authentication
  - Only the owner of the private key knows it, so text enciphered with private key must have been generated by the owner

More Security Services

- Integrity
  - Enciphered letters cannot be changed undetectably without knowing private key
- Non-Repudiation
  - Message enciphered with private key came from someone who knew it

Warnings

- Encipher message in blocks considerably larger than the examples here
  - If 1 character per block, RSA can be broken using statistical attacks (just like classical cryptosystems)
  - Attacker cannot alter letters, but can rearrange them and alter message meaning
    - Example: reverse enciphered message of text ON to get NO

Cryptographic Checksums

- Mathematical function to generate a set of k bits from a set of n bits (where $k \leq n$).
  - k is smaller then n except in unusual circumstances
- Example: ASCII parity bit
  - ASCII has 7 bits; 8th bit is “parity”
  - Even parity: even number of 1 bits
  - Odd parity: odd number of 1 bits

Example Use

- Bob receives “10111101” as bits.
  - Sender is using even parity; 6 1 bits, so character was received correctly
    - Note: could be garbled, but 2 bits would need to have been changed to preserve parity
  - Sender is using odd parity; even number of 1 bits, so character was not received correctly

Definition

- Cryptographic checksum $h: A \rightarrow B$:
  1. For any $x \in A$, $h(x)$ is easy to compute
  2. For any $y \in B$, it is computationally infeasible to find $x \in A$ such that $h(x) = y$
  3. It is computationally infeasible to find two inputs x, $x' \in A$ such that $x \neq x'$ and $h(x) = h(x')$
     – Alternate form (stronger): Given any $x \in A$, it is computationally infeasible to find a different $x' \in A$ such that $h(x) = h(x')$.

Collisions

- If $x \neq x'$ and $h(x) = h(x')$, x and x' are a *collision*
  - Pigeonhole principle: if there are n containers for $n+1$ objects, then at least one container will have 2 objects in it.
  - Application: if there are 32 files and 8 possible cryptographic checksum values, at least one value corresponds to at least 4 files

Keys

- Keyed cryptographic checksum: requires cryptographic key
  - DES in chaining mode: encipher message, use last n bits. Requires a key to encipher, so it is a keyed cryptographic checksum.
- Keyless cryptographic checksum: requires no cryptographic key
  - MD5 and SHA-1 are best known; others include MD4, HAVAL, and Snefru

HMAC

- Make keyed cryptographic checksums from keyless cryptographic checksums
- *h* keyless cryptographic checksum function that takes data in blocks of *b* bytes and outputs blocks of *l* bytes. k' is cryptographic key of length *b* bytes
  - If short, pad with 0 bytes; if long, hash to length *b*
- *ipad* is 00110110 repeated *b* times
- *opad* is 01011100 repeated *b* times
- HMAC-$h(k, m) = h(k' \oplus opad \| h(k' \oplus ipad \| m))$
  - $\oplus$ exclusive or, $\|$ concatenation

Key Points

- Two main types of cryptosystems: classical and public key
- Classical cryptosystems encipher and decipher using the same key
  - Or one key is easily derived from the other
- Public key cryptosystems encipher and decipher using different keys
  - Computationally infeasible to derive one from the other
- Cryptographic checksums provide a check on integrity

Chapter 3

Traditional Symmetric-Key Ciphers

Objectives

- To define the terms and the concepts of symmetric key ciphers
- To emphasize the two categories of traditional ciphers: substitution and transposition ciphers
- To describe the categories of cryptanalysis used to break the symmetric ciphers
- To introduce the concepts of the stream ciphers and block ciphers
- To discuss some very dominant ciphers used in the past, such as the Enigma machine

3-1 INTRODUCTION

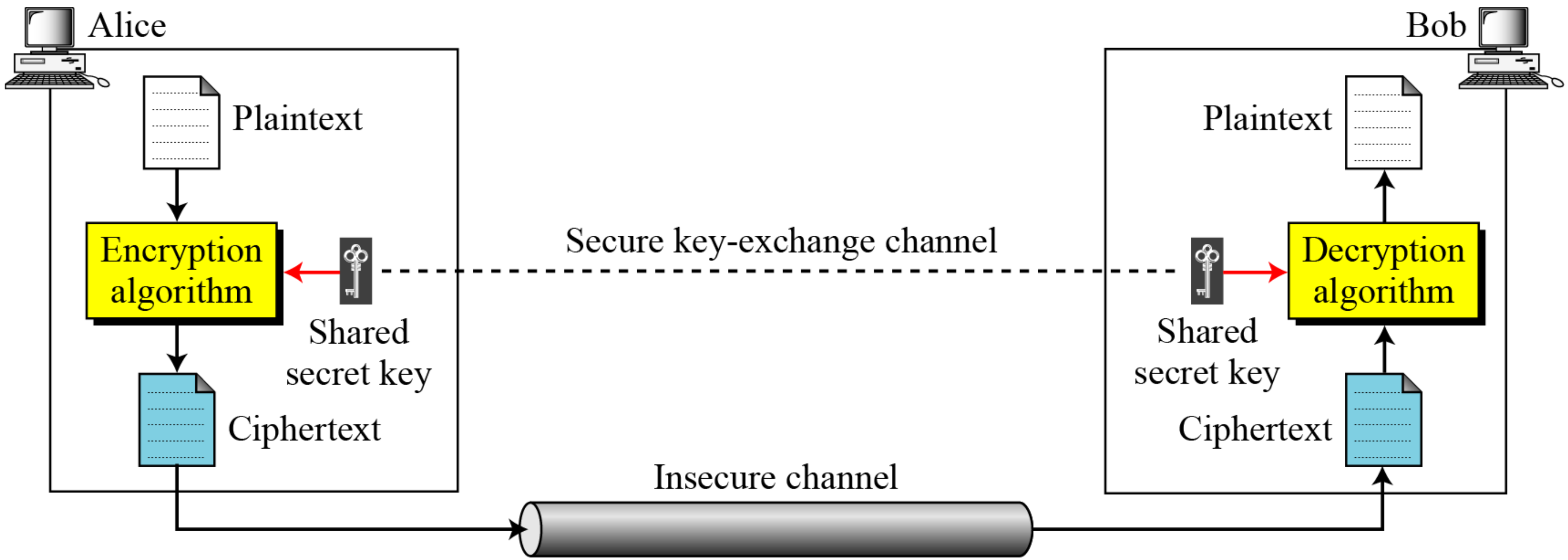
Figure 3.1 shows the general idea behind a symmetric-key cipher. The original message from Alice to Bob is called plaintext; the message that is sent through the channel is called the ciphertext. To create the ciphertext from the plaintext, Alice uses an encryption algorithm and a shared secret key. To create the plaintext from ciphertext, Bob uses a decryption algorithm and the same secret key.

Topics discussed in this section:

3.1.1 Kerckhoff's Principle
3.1.2 Cryptanalysis
3.1.3 Categories of Traditional Ciphers

3.1 Continued

Figure 3.1 ***General idea of symmetric-key cipher***



3.1 Continued

If P is the plaintext, C is the ciphertext, and K is the key,

Encryption: $C = E_k(P)$ Decryption: $P = D_k(C)$

In which, $D_k(E_k(x)) = E_k(D_k(x)) = x$

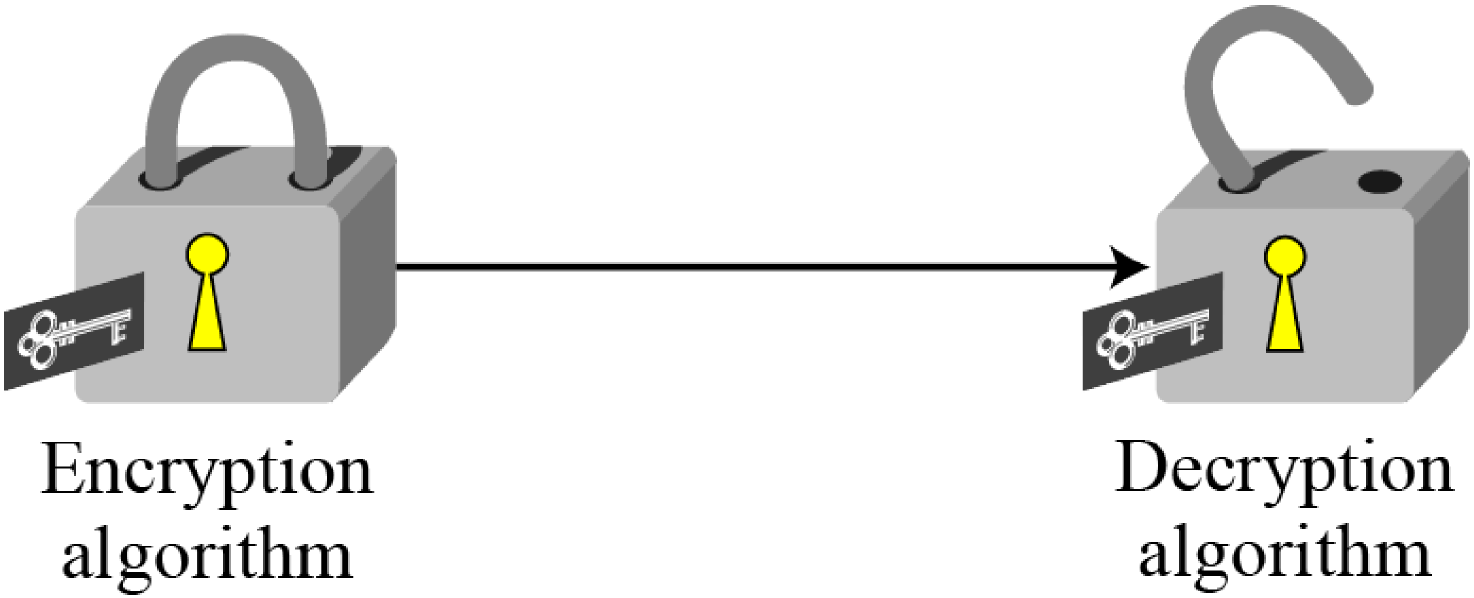
We assume that Bob creates $\mathbf{P_1}$***; we prove that*** $\mathbf{P_1 = P}$**:**

Alice: $C = E_k(P)$

Bob: $P_1 = D_k(C) = D_k(E_k(P)) = P$

3.1 Continued

Figure 3.2 ***Locking and unlocking with the same key***



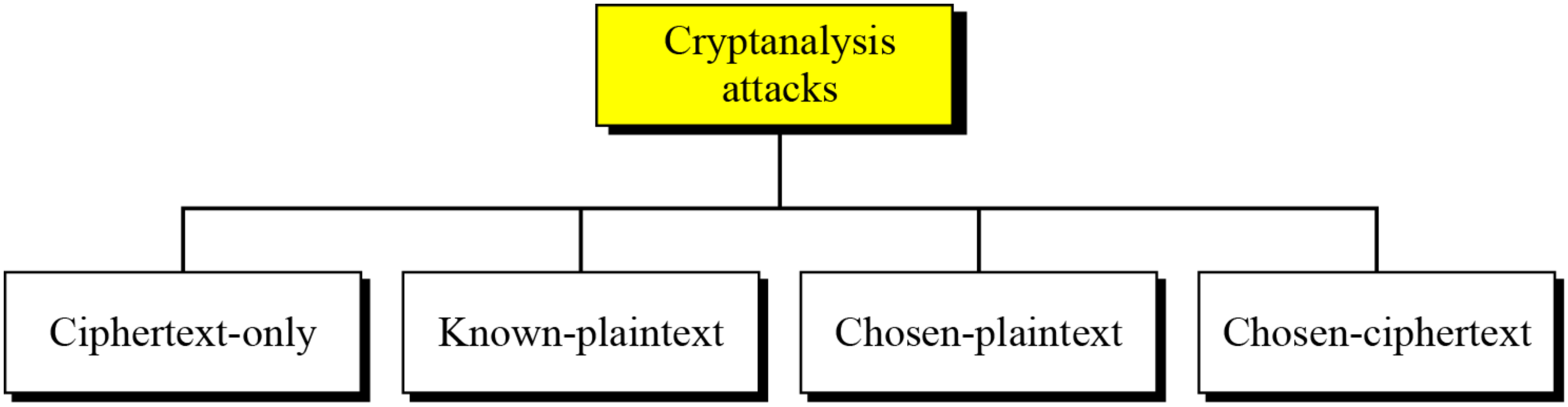
3.1.1 Kerckhoff's Principle

Based on Kerckhoff's principle, one should always assume that the adversary, Eve, knows the encryption/decryption algorithm. The resistance of the cipher to attack must be based only on the secrecy of the key.

3.1.2 Cryptanalysis

As cryptography is the science and art of creating secret codes, cryptanalysis is the science and art of breaking those codes.

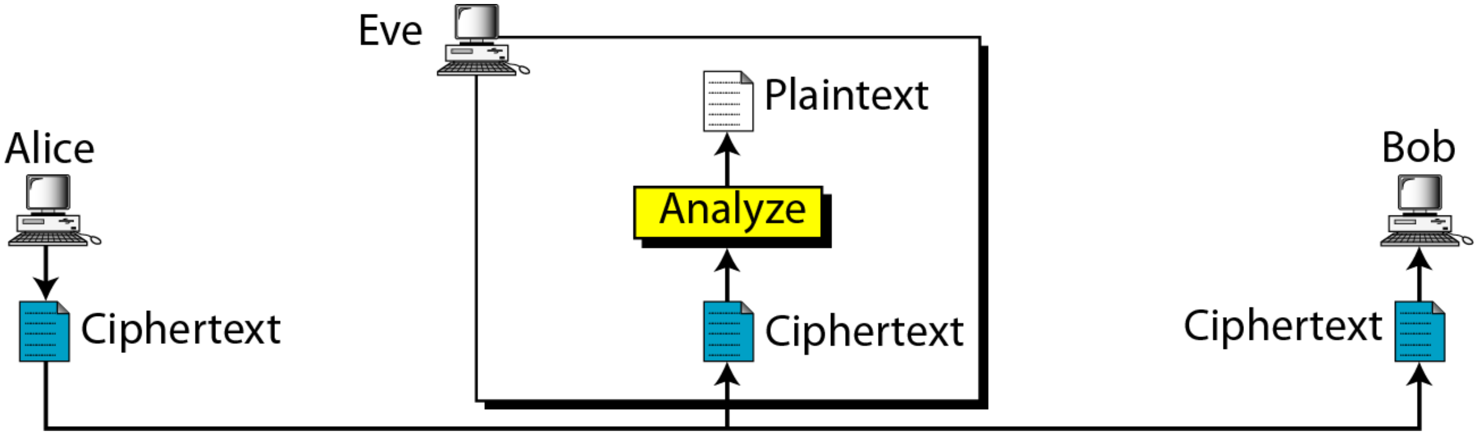
Figure 3.3 ***Cryptanalysis attacks***



3.1.2 Continued

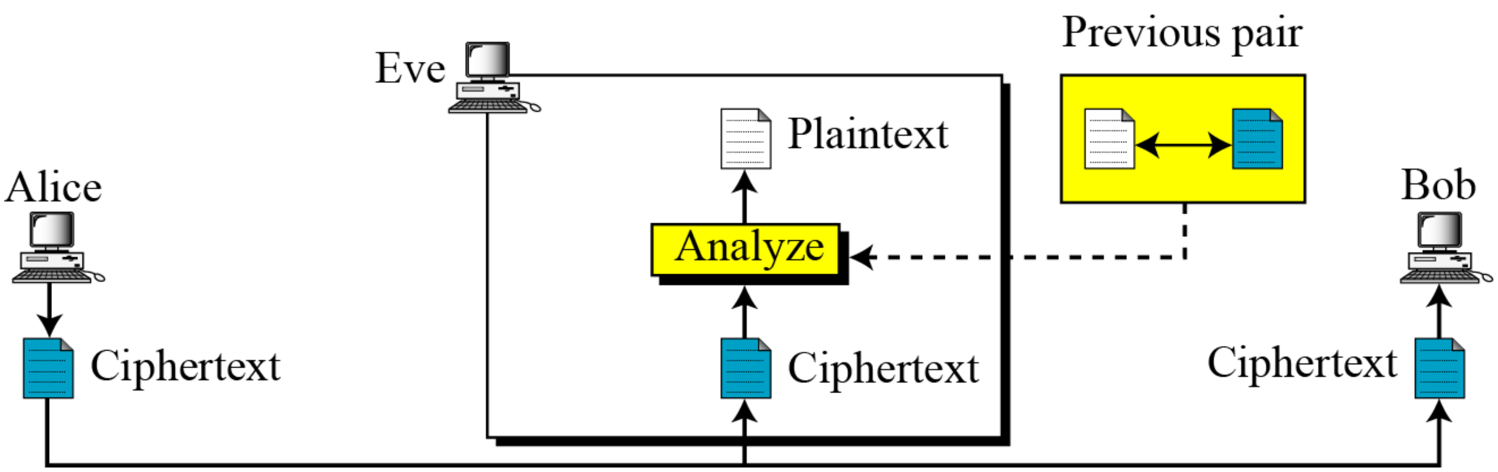
Ciphertext-Only Attack

Figure 3.4 ***Ciphertext-only attack***



Known-Plaintext Attack

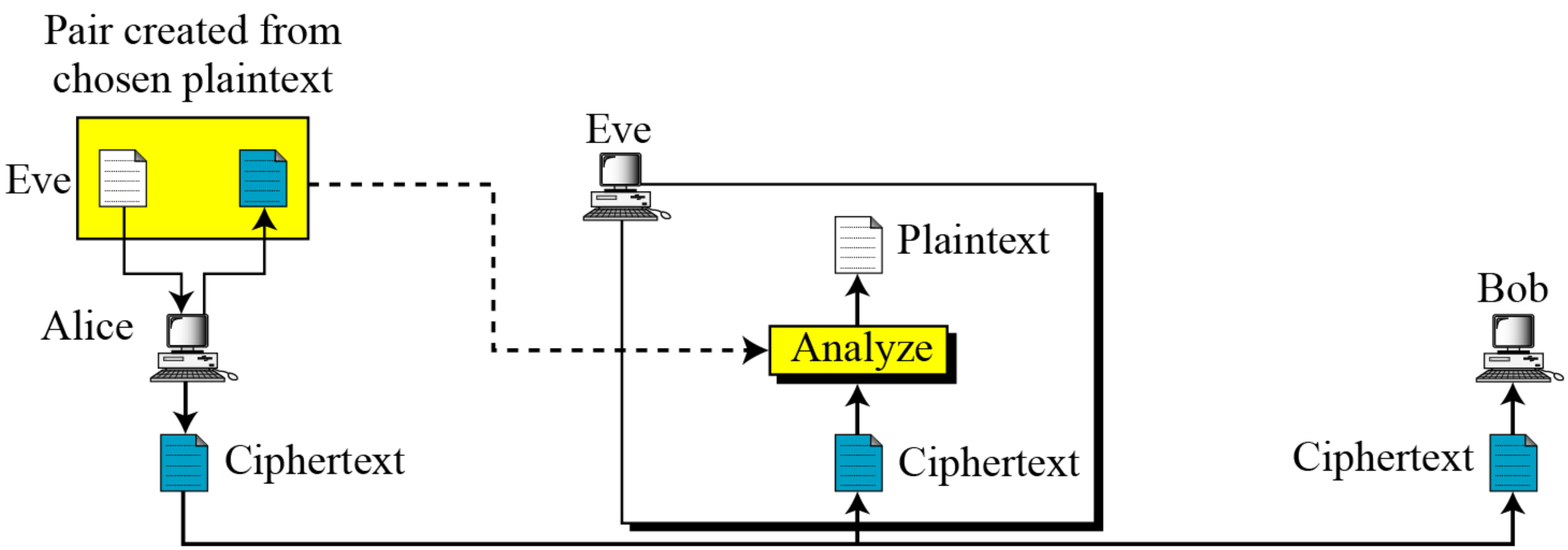
Figure 3.5 ***Known-plaintext attack***



3.1.2 Continued

Chosen-Plaintext Attack

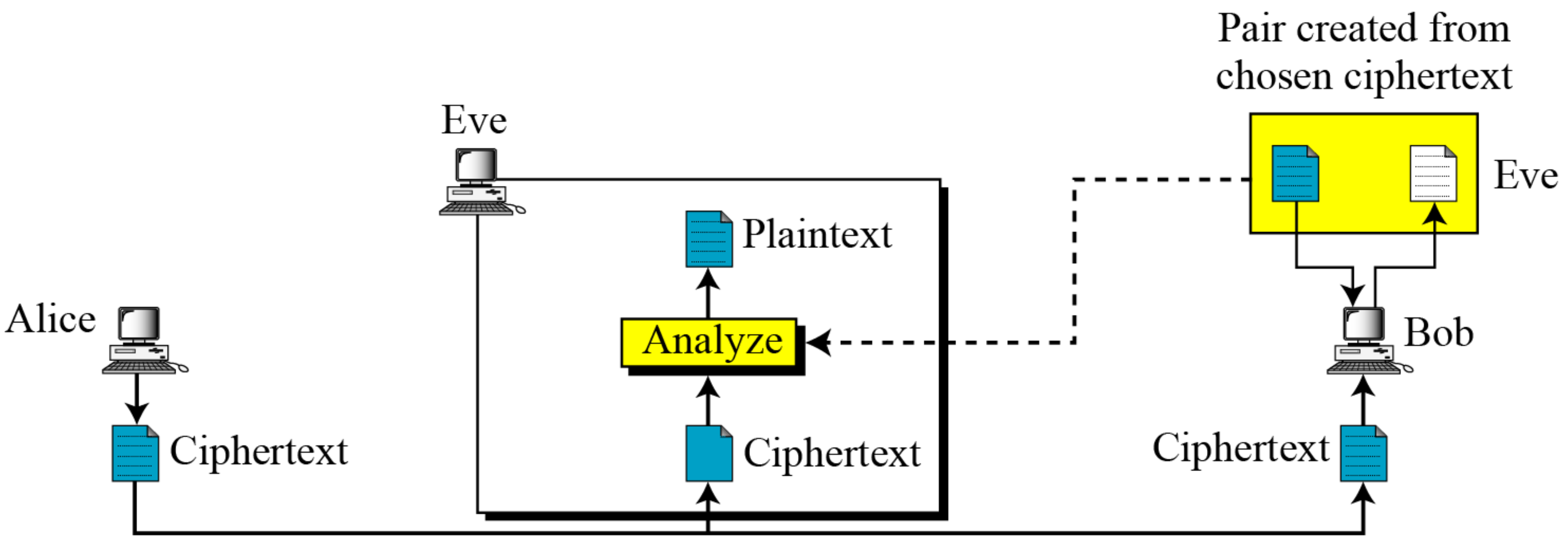
Figure 3.6 ***Chosen-plaintext attack***



3.1.2 Continued

Chosen-Ciphertext Attack

Figure 3.7 ***Chosen-ciphertext attack***



3-2 SUBSTITUTION CIPHERS

A substitution cipher replaces one symbol with another. Substitution ciphers can be categorized as either monoalphabetic ciphers or polyalphabetic ciphers.

Note

A substitution cipher replaces one symbol with another.

Topics discussed in this section:

3.2.1 Monoalphabetic Ciphres
3.2.2 Polyalphabetic Ciphers

3.2.1 Monoalphabetic Ciphers

Note

In monoalphabetic substitution, the relationship between a symbol in the plaintext to a symbol in the ciphertext is always one-to-one.

3.2.1 Continued

Example 3.1

The following shows a plaintext and its corresponding ciphertext. The cipher is probably monoalphabetic because both *l*'s (els) are encrypted as *O*'s.

Plaintext: hello **Ciphertext:** KHOOR

Example 3.2

The following shows a plaintext and its corresponding ciphertext. The cipher is not monoalphabetic because each *l* (el) is encrypted by a different character.

Plaintext: hello **Ciphertext:** KHOOR

Additive Cipher

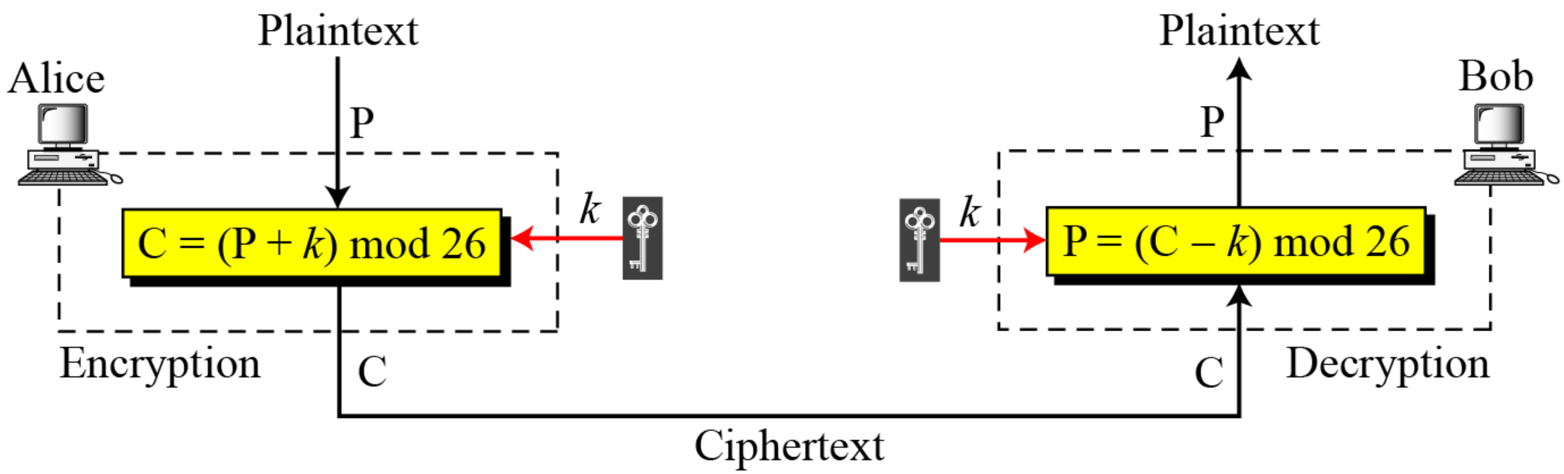
The simplest monoalphabetic cipher is the additive cipher. This cipher is sometimes called a shift cipher and sometimes a Caesar cipher, but the term additive cipher better reveals its mathematical nature.

Figure 3.8 *Plaintext and ciphertext in Z_{26}*

Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value →	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

3.2.1 Continued

Figure 3.9 *Additive cipher*



Note

When the cipher is additive, the plaintext, ciphertext, and key are integers in Z_{26}.

3.2.1 Continued

Example 3.3

Use the additive cipher with key = 15 to encrypt the message "hello".

Solution

We apply the encryption algorithm to the plaintext, character by character:

Plaintext: h → 07	Encryption: (07 + 15) mod 26	Ciphertext: 22 → W
Plaintext: e → 04	Encryption: (04 + 15) mod 26	Ciphertext: 19 → T
Plaintext: l → 11	Encryption: (11 + 15) mod 26	Ciphertext: 00 → A
Plaintext: l → 11	Encryption: (11 + 15) mod 26	Ciphertext: 00 → A
Plaintext: o → 14	Encryption: (14 + 15) mod 26	Ciphertext: 03 → D

Example 3.4

Use the additive cipher with key = 15 to decrypt the message "WTAAD".

Solution

We apply the decryption algorithm to the plaintext character by character:

Ciphertext: W $\rightarrow$ 22	Decryption: (22 – 15) mod 26	Plaintext: 07 $\rightarrow$ h
Ciphertext: T $\rightarrow$ 19	Decryption: (19 – 15) mod 26	Plaintext: 04 $\rightarrow$ e
Ciphertext: A $\rightarrow$ 00	Decryption: (00 – 15) mod 26	Plaintext: 11 $\rightarrow$ l
Ciphertext: A $\rightarrow$ 00	Decryption: (00 – 15) mod 26	Plaintext: 11 $\rightarrow$ l
Ciphertext: D $\rightarrow$ 03	Decryption: (03 – 15) mod 26	Plaintext: 14 $\rightarrow$ o

3.2.1 Continued

Shift Cipher and Caesar Cipher

Historically, additive ciphers are called shift ciphers. Julius Caesar used an additive cipher to communicate with his officers. For this reason, additive ciphers are sometimes referred to as the Caesar cipher. Caesar used a key of 3 for his communications.

Note

Additive ciphers are sometimes referred to as shift ciphers or Caesar cipher.

3.2.1 Continued

Example 3.5

Eve has intercepted the ciphertext "UVACLYFZLJBYL". Show how she can use a brute-force attack to break the cipher.

Solution

Eve tries keys from 1 to 7. With a key of 7, the plaintext is "not very secure", which makes sense.

Ciphertext: UVACLYFZLJBYL

K = 1	→	**Plaintext:** tuzbkxeykiaxk
K = 2	→	**Plaintext:** styajwdxjhzwj
K = 3	→	**Plaintext:** rsxzivcwigyvi
K = 4	→	**Plaintext:** qrwyhubvhfxuh
K = 5	→	**Plaintext:** pqvxgtaugewtg
K = 6	→	**Plaintext:** opuwfsztfdvsf
K = 7	→	**Plaintext:** notverysecure

3.2.1 Continued

Table 3.1 *Frequency of characters in English*

Letter	*Frequency*	*Letter*	*Frequency*	*Letter*	*Frequency*	*Letter*	*Frequency*
E	12.7	H	6.1	W	2.3	K	0.08
T	9.1	R	6.0	F	2.2	J	0.02
A	8.2	D	4.3	G	2.0	Q	0.01
O	7.5	L	4.0	Y	2.0	X	0.01
I	7.0	C	2.8	P	1.9	Z	0.01
N	6.7	U	2.8	B	1.5		
S	6.3	M	2.4	V	1.0		

Table 3.2 *Frequency of diagrams and trigrams*

Digram	TH, HE, IN, ER, AN, RE, ED, ON, ES, ST, EN, AT, TO, NT, HA, ND, OU, EA, NG, AS, OR, TI, IS, ET, IT, AR, TE, SE, HI, OF
Trigram	THE, ING, AND, HER, ERE, ENT, THA, NTH, WAS, ETH, FOR, DTH

3.2.1 *Continued*

Example 3.6

Eve has intercepted the following ciphertext. Using a statistical attack, find the plaintext.

XLILSYWIMWRSAJSVWEPIJSVJSYVQMPPMSRHSPPEVWMXMWASVX-LQSVILY-VVCFIJSVIXLIWIPPIVVIGIMZIWQSVISJJIVW

Solution

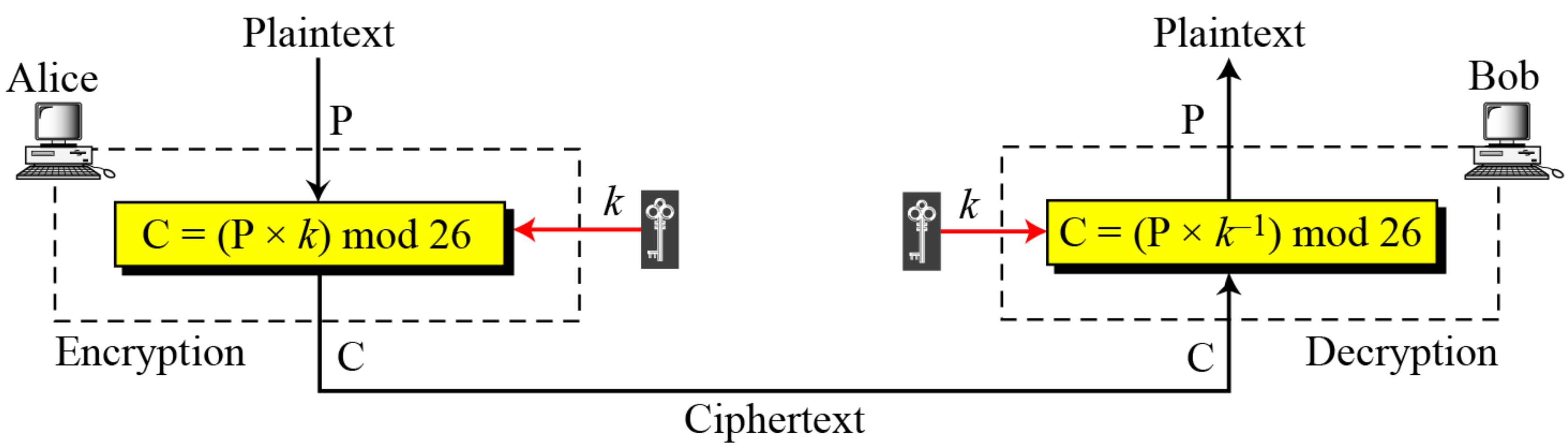
When Eve tabulates the frequency of letters in this ciphertext, she gets: I =14, V =13, S =12, and so on. The most common character is I with 14 occurrences. This means key = 4.

the house is now for sale for four million dollars it is worth more hurry before the seller receives more offers

3.2.1 Continued

Multiplicative Ciphers

Figure 3.10 ***Multiplicative cipher***



Note

In a multiplicative cipher, the plaintext and ciphertext are integers in Z_{26}; the key is an integer in Z_{26}*.

Example 3.7

What is the key domain for any multiplicative cipher?

Solution

The key needs to be in Z_{26}*. This set has only 12 members: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25.

Example 3.8

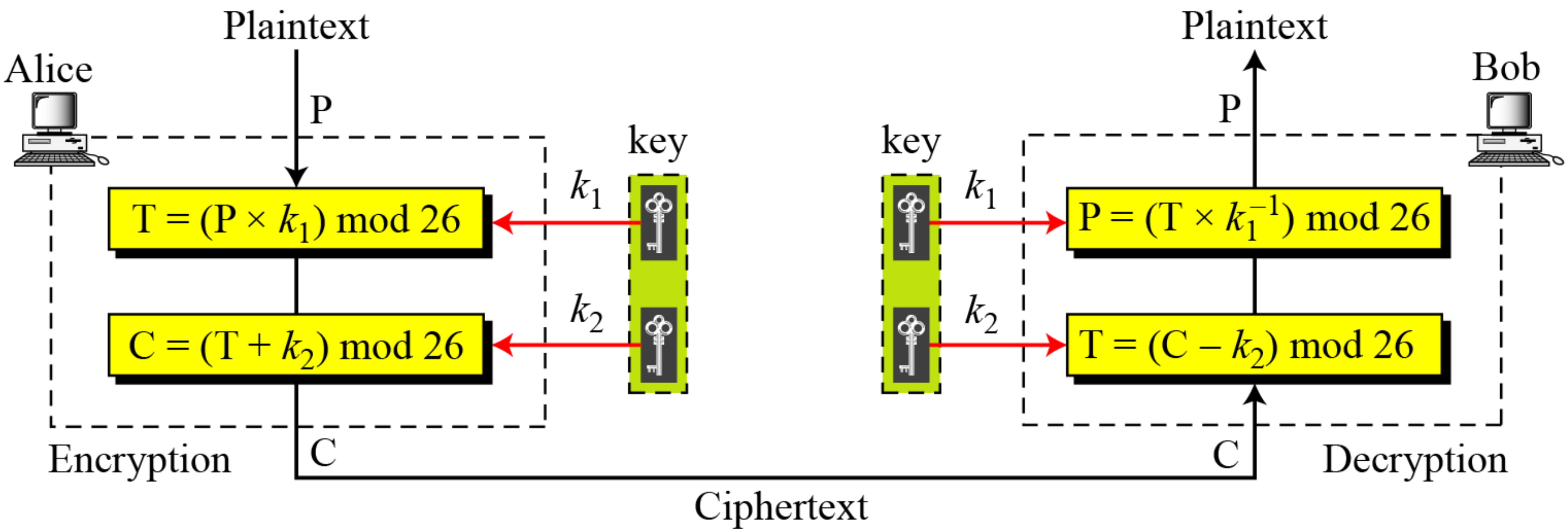
We use a multiplicative cipher to encrypt the message "hello" with a key of 7. The ciphertext is "XCZZU".

Plaintext: h → 07	Encryption: (07 × 07) mod 26	ciphertext: 23 → X
Plaintext: e → 04	Encryption: (04 × 07) mod 26	ciphertext: 02 → C
Plaintext: l → 11	Encryption: (11 × 07) mod 26	ciphertext: 25 → Z
Plaintext: l → 11	Encryption: (11 × 07) mod 26	ciphertext: 25 → Z
Plaintext: o → 14	Encryption: (14 × 07) mod 26	ciphertext: 20 → U

3.2.1 Continued

Affine Ciphers

Figure 3.11 ***Affine cipher***



$$C = (P \times k_1 + k_2) \bmod 26 \qquad P = ((C - k_2) \times k_1^{-1}) \bmod 26$$

where k_1^{-1} is the multiplicative inverse of k_1 and $-k_2$ is the additive inverse of k_2

Example 3.09

The affine cipher uses a pair of keys in which the first key is from Z_{26}* and the second is from Z_{26}. The size of the key domain is 26 × 12 = 312.

Example 3.10

Use an affine cipher to encrypt the message "hello" with the key pair (7, 2).

P: h → 07	Encryption: $(07 \times 7 + 2) \bmod 26$	C: 25 → Z
P: e → 04	Encryption: $(04 \times 7 + 2) \bmod 26$	C: 04 → E
P: l → 11	Encryption: $(11 \times 7 + 2) \bmod 26$	C: 01 → B
P: l → 11	Encryption: $(11 \times 7 + 2) \bmod 26$	C: 01 → B
P: o → 14	Encryption: $(14 \times 7 + 2) \bmod 26$	C: 22 → W

Example 3.11

Use the affine cipher to decrypt the message “ZEBBW” with the key pair (7, 2) in modulus 26.

Solution

C: Z $\rightarrow$ 25	Decryption: $((25 - 2) \times 7^{-1}) \bmod 26$	P:07 $\rightarrow$ h
C: E $\rightarrow$ 04	Decryption: $((04 - 2) \times 7^{-1}) \bmod 26$	P:04 $\rightarrow$ e
C: B $\rightarrow$ 01	Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$	P:11 $\rightarrow$ l
C: B $\rightarrow$ 01	Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$	P:11 $\rightarrow$ l
C: W $\rightarrow$ 22	Decryption: $((22 - 2) \times 7^{-1}) \bmod 26$	P:14 $\rightarrow$ o

Example 3.12

The additive cipher is a special case of an affine cipher in which $k_1 = 1$. The multiplicative cipher is a special case of affine cipher in which $k_2 = 0$.

Monoalphabetic Substitution Cipher

Because additive, multiplicative, and affine ciphers have small key domains, they are very vulnerable to brute-force attack.

A better solution is to create a mapping between each plaintext character and the corresponding ciphertext character. Alice and Bob can agree on a table showing the mapping for each character.

Figure 3.12 *An example key for monoalphabetic substitution cipher*

Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	N	O	A	T	R	B	E	C	F	U	X	D	Q	G	Y	L	K	H	V	I	J	M	P	Z	S	W

3.2.1 *Continued*

Example 3.13

We can use the key in Figure 3.12 to encrypt the message

this message is easy to encrypt but hard to find the key

The ciphertext is

ICFVQRVVNEFVRNVSIYRGAHSLIOJICNHTIYBFGTICRXRS

3.2.2 Polyalphabetic Ciphers

In polyalphabetic substitution, each occurrence of a character may have a different substitute. The relationship between a character in the plaintext to a character in the ciphertext is one-to-many.

Autokey Cipher

$P = P_1P_2P_3 \ldots$ $C = C_1C_2C_3 \ldots$ $k = (k_1, P_1, P_2, \ldots)$

Encryption: $C_i = (P_i + k_i) \bmod 26$ Decryption: $P_i = (C_i - k_i) \bmod 26$

Example 3.14

Assume that Alice and Bob agreed to use an autokey cipher with initial key value k_1 = 12. Now Alice wants to send Bob the message "Attack is today". Enciphering is done character by character.

Plaintext:	a	t	t	a	c	k	i	s	t	o	d	a	y
P's Values:	00	19	19	00	02	10	08	18	19	14	03	00	24
Key stream:	*12*	*00*	*19*	*19*	*00*	*02*	*10*	*08*	*18*	*19*	*14*	*03*	*00*
C's Values:	12	19	12	19	02	12	18	00	11	7	17	03	24
Ciphertext:	**M**	**T**	**M**	**T**	**C**	**M**	**S**	**A**	**L**	**H**	**R**	**D**	**Y**

Playfair Cipher

Figure 3.13 *An example of a secret key in the Playfair cipher*

Secret Key =

L	G	D	B	A
Q	M	H	E	C
U	R	N	I/J	F
X	V	S	O	K
Z	Y	W	T	P

Example 3.15

Let us encrypt the plaintext "hello" using the key in Figure 3.13.

he → EC   lx → QZ   lo → BX

Plaintext: hello   Ciphertext: ECQZBX

Vigenere Cipher

$P = P_1P_2P_3 \ldots$ $C = C_1C_2C_3 \ldots$ $K = [(k_1, k_2, \ldots, k_m), (k_1, k_2, \ldots, k_m), \ldots]$

Encryption: $C_i = P_i + k_i$ Decryption: $P_i = C_i - k_i$

Example 3.16

We can encrypt the message "She is listening" using the 6-character keyword "PASCAL".

Plaintext:	s	h	e	i	s	l	i	s	t	e	n	i	n	g
P's values:	18	07	04	08	18	11	08	18	19	04	13	08	13	06
Key stream:	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***
C's values:	**07**	**07**	**22**	**10**	**18**	**22**	**23**	**18**	**11**	**6**	**13**	**19**	**02**	**06**
Ciphertext:	**H**	**H**	**W**	**K**	**S**	**W**	**X**	**S**	**L**	**G**	**N**	**T**	**C**	**G**

Example 3.16

Let us see how we can encrypt the message "She is listening" using the 6-character keyword "PASCAL". The initial key stream is (15, 0, 18, 2, 0, 11). The key stream is the repetition of this initial key stream (as many times as needed).

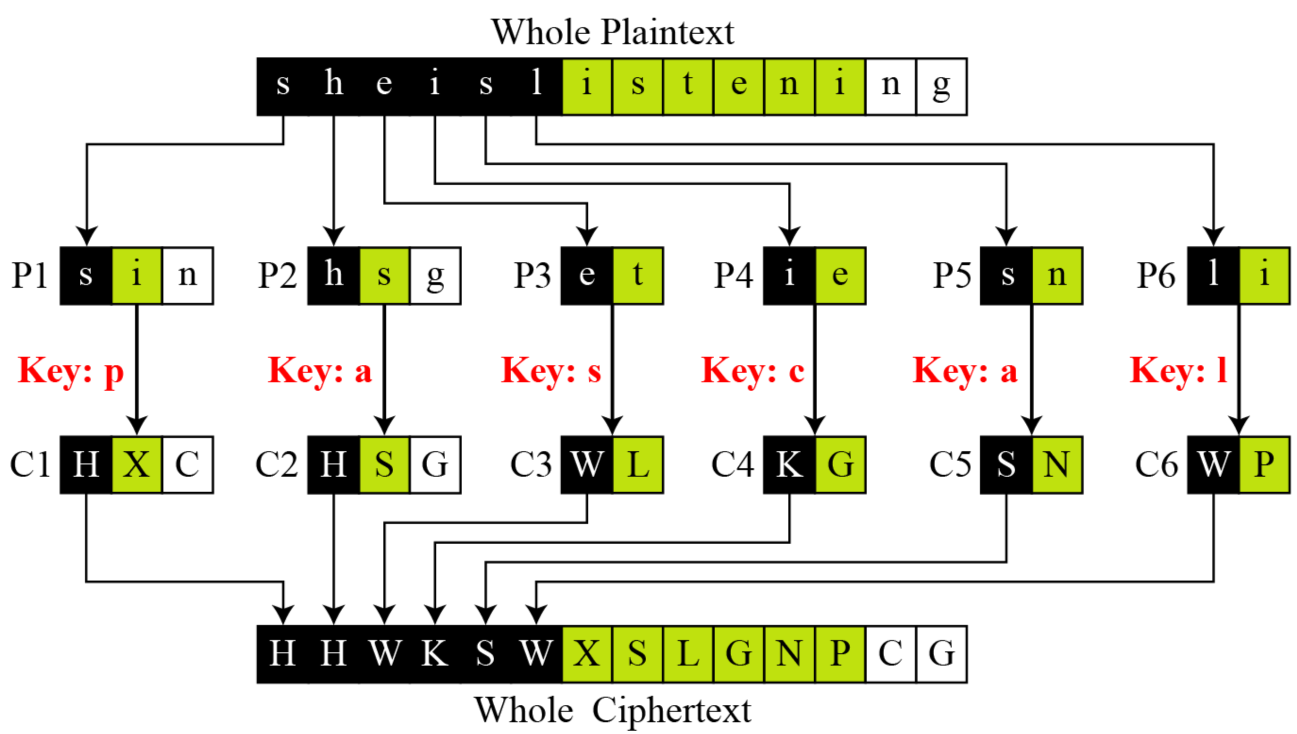
Plaintext:	s	h	e	i	s	l	i	s	t	e	n	i	n	g
P's values:	18	07	04	08	18	11	08	18	19	04	13	08	13	06
Key stream:	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***
C's values:	**07**	**07**	**22**	**10**	**18**	**22**	**23**	**18**	**11**	**6**	**13**	**19**	**02**	**06**
Ciphertext:	**H**	**H**	**W**	**K**	**S**	**W**	**X**	**S**	**L**	**G**	**N**	**T**	**C**	**G**

3.2.2 *Continued*

Example 3.17

Vigenere cipher can be seen as combinations of m additive ciphers.

Figure 3.14 *A Vigenere cipher as a combination of m additive ciphers*



3.2.2 *Continued*

Example 3.18

Using Example 3.18, we can say that the additive cipher is a special case of Vigenere cipher in which *m* = 1.

Table 3.3

A Vigenere Tableau

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	v	v	w	x	y	z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

3.2.2 Continued

Vigenere Cipher (Crypanalysis)

Example 3.19

Let us assume we have intercepted the following ciphertext:

LIOMWGFEGGDVWGHHCQUCRHRWAGWIOWQLKGZETKKMEVLWPCZVGTH-
VTSGXQOVGCSVETQLTJSUMVWVEUVLXEWSLGFZMVVWLGYHCUSWXQH-
KVGSHEEVFLCFDGVSUMPHKIRZDMPHHBVWVWJWIXGFWLTSHGJOUEEHH-
VUCFVGOWICQLTJSUXGLW

The Kasiski test for repetition of three-character segments yields the results shown in Table 3.4.

String	*First Index*	*Second Index*	*Difference*
JSU	68	168	100
SUM	69	117	48
VWV	72	132	60
MPH	119	127	8

Example 3.19

Let us assume we have intercepted the following ciphertext:

LIOMWGFEGGDVWGHHCQUCRHRWAGWIOWQLKGZETKKMEVLWPCZVGTH-
VTSGXQOVGCSVETQLTJSUMVWVEUVLXEWSLGFZMVVWLGYHCUSWXQH-
KVGSHEEVFLCFDGVSUMPHKIRZDMPHHBVWVWJWIXGFWLTSHGJOUEEHH-
VUCFVGOWICQLTJSUXGLW

The Kasiski test for repetition of three-character segments yields the results shown in Table 3.4.

String	*First Index*	*Second Index*	*Difference*
JSU	68	168	100
SUM	69	117	48
VWV	72	132	60
MPH	119	127	8

3.2.2 Continued

Example 3.19 (Continued)

The greatest common divisor of differences is 4, which means that the key length is multiple of 4. First try *m* = 4.

```
C1: LWGWCRAOKTEPGTQCTJVUEGVGUQGECVPRPVJGTJEUGCJG
P1: jueuapymircneroarhtsthihytrahcieixsthcarrehe
C2: IGGGQHGWGKVCTSOSQSWVWFVYSHSVFSHZHWWFSOHCOQSL
P2: ussscts iswhofeaeceihcetesoecatnpntherhctecex
C3: OFDHURWQZKLZHGVVLUVLSZWHWKHFDUKDHVIWHUHFWLUW
P3: lcaerotnwhiwedssirsiirhketehretltiideatrairt
C4: MEVHCWILEMWVVXGETMEXLMLCXVELGMIMBWXLGEVVITX
P4: iardysehaisrrtcapiafpwtethecarhaesfterectpt
```

In this case, the plaintext makes sense.

Julius Caesar used a cryptosystem in his wars, which is now referred to as Caesar cipher. It is an additive cipher with the key set to three. Each character in the plaintext is shifted three characters to create ciphertext.

Hill Cipher

Figure 3.15 *Key in the Hill cipher*

$$K = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1m} \\ k_{21} & k_{22} & \cdots & k_{2m} \\ \vdots & \vdots & & \vdots \\ k_{m1} & k_{m2} & \cdots & k_{mm} \end{bmatrix}$$

$$C_1 = P_1 k_{11} + P_2 k_{21} + \cdots + P_m k_{m1}$$
$$C_2 = P_1 k_{12} + P_2 k_{22} + \cdots + P_m k_{m2}$$
$$\cdots$$
$$C_m = P_1 k_{1m} + P_2 k_{2m} + \cdots + P_m k_{mm}$$

Note

The key matrix in the Hill cipher needs to have a multiplicative inverse.

3.2.2 *Continued*

Example 3.20

For example, the plaintext "code is ready" can make a 3 × 4 matrix when adding extra bogus character "z" to the last block and removing the spaces. The ciphertext is "OHKNIHGKLISS".

Figure 3.16 ***Example 3.20***

$$\overset{C}{\begin{bmatrix} 14 & 07 & 10 & 13 \\ 08 & 07 & 06 & 11 \\ 11 & 08 & 18 & 18 \end{bmatrix}} = \overset{P}{\begin{bmatrix} 02 & 14 & 03 & 04 \\ 08 & 18 & 17 & 04 \\ 00 & 03 & 24 & 25 \end{bmatrix}} \overset{K}{\begin{bmatrix} 09 & 07 & 11 & 13 \\ 04 & 07 & 05 & 06 \\ 02 & 21 & 14 & 09 \\ 03 & 23 & 21 & 08 \end{bmatrix}}$$

a. Encryption

$$\overset{P}{\begin{bmatrix} 02 & 14 & 03 & 04 \\ 08 & 18 & 17 & 04 \\ 00 & 03 & 24 & 25 \end{bmatrix}} = \overset{C}{\begin{bmatrix} 14 & 07 & 10 & 13 \\ 08 & 07 & 06 & 11 \\ 11 & 08 & 18 & 18 \end{bmatrix}} \overset{K^{-1}}{\begin{bmatrix} 02 & 15 & 22 & 03 \\ 15 & 00 & 19 & 03 \\ 09 & 09 & 03 & 11 \\ 17 & 00 & 04 & 07 \end{bmatrix}}$$

b. Decryption

Example 3.21

Assume that Eve knows that *m* = 3. She has intercepted three plaintext/ciphertext pair blocks (not necessarily from the same message) as shown in Figure 3.17.

Figure 3.17 *Example 3.21*

$$\begin{bmatrix} 05 & 07 & 10 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 03 & 06 & 00 \end{bmatrix}$$

$$\begin{bmatrix} 13 & 17 & 07 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 14 & 16 & 09 \end{bmatrix}$$

$$\begin{bmatrix} 00 & 05 & 04 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 03 & 17 & 11 \end{bmatrix}$$

P C

3.2.2 Continued

Example 3.21 (Continued)

She makes matrices P and C from these pairs. Because P is invertible, she inverts the P matrix and multiplies it by C to get the K matrix as shown in Figure 3.18.

Figure 3.18 ***Example 3.21***

$$\begin{bmatrix} 02 & 03 & 07 \\ 05 & 07 & 09 \\ 01 & 02 & 11 \end{bmatrix} = \begin{bmatrix} 21 & 14 & 01 \\ 00 & 08 & 25 \\ 13 & 03 & 08 \end{bmatrix} \begin{bmatrix} 03 & 06 & 00 \\ 14 & 16 & 09 \\ 03 & 17 & 11 \end{bmatrix}$$

$$\mathrm{K} \qquad \mathrm{P}^{-1} \qquad \mathrm{C}$$

Now she has the key and can break any ciphertext encrypted with that key.

3.2.2 Continued

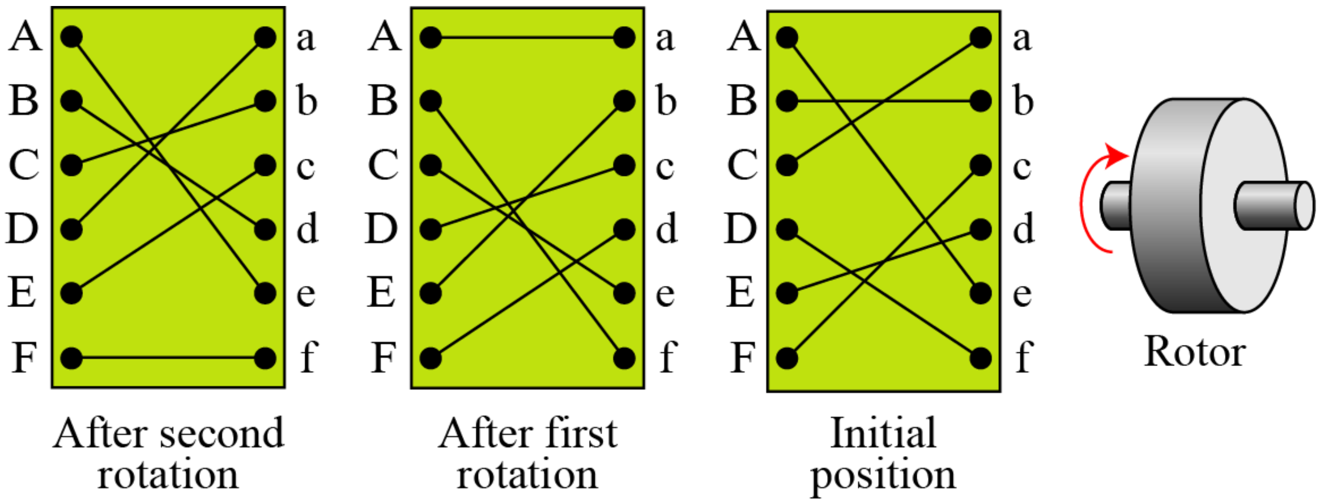
One-Time Pad

One of the goals of cryptography is perfect secrecy. A study by Shannon has shown that perfect secrecy can be achieved if each plaintext symbol is encrypted with a key randomly chosen from a key domain. This idea is used in a cipher called one-time pad, invented by Vernam.

3.2.2 *Continued*

Rotor Cipher

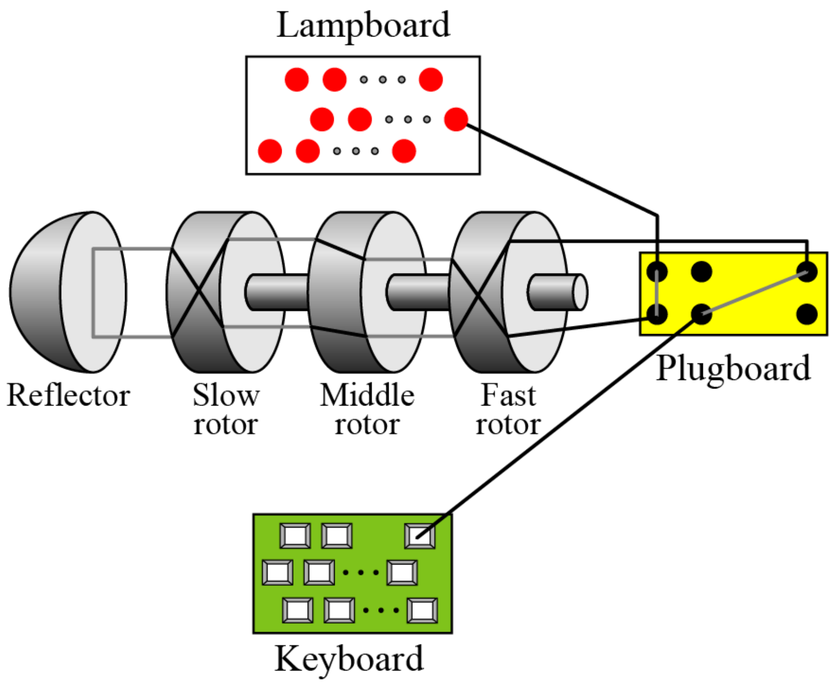
Figure 3.19 ***A rotor cipher***



3.2.2 *Continued*

Enigma Machine

Figure 3.20 *A schematic of the Enigma machine*



3-3 TRANSPOSITION CIPHERS

A transposition cipher does not substitute one symbol for another, instead it changes the location of the symbols.

Note

A transposition cipher reorders symbols.

Topics discussed in this section:

3.3.1 Keyless Transposition Ciphers
3.3.2 Keyed Transposition Ciphers
3.3.3 Combining Two Approaches

3.3.1 Keyless Transposition Ciphers

Simple transposition ciphers, which were used in the past, are keyless.

Example 3.22

A good example of a keyless cipher using the first method is the rail fence cipher. The ciphertext is created reading the pattern row by row. For example, to send the message "Meet me at the park" to Bob, Alice writes

```
m       e       m       a       t       e       a       k
  ↘   ↗   ↘   ↗   ↘   ↗   ↘   ↗   ↘   ↗   ↘   ↗   ↘   ↗
    e       t       e       t       h       p       r
```

She then creates the ciphertext "MEMATEAKETETHPR".

Example 3.23

Alice and Bob can agree on the number of columns and use the second method. Alice writes the same plaintext, row by row, in a table of four columns.

m	e	e	t
m	e	a	t
t	h	e	p
a	r	k	

She then creates the ciphertext **"MMTAEEHREAEKTTP"**.

3.3.1 *Continued*

Example 3.24

The cipher in Example 3.23 is actually a transposition cipher. The following shows the permutation of each character in the plaintext into the ciphertext based on the positions.

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
01	05	09	13	02	06	10	13	03	07	11	15	04	08	12

The second character in the plaintext has moved to the fifth position in the ciphertext; the third character has moved to the ninth position; and so on. Although the characters are permuted, there is a pattern in the permutation: (01, 05, 09, 13), (02, 06, 10, 13), (03, 07, 11, 15), and (08, 12). In each section, the difference between the two adjacent numbers is 4.

3.3.2 Keyed Transposition Ciphers

The keyless ciphers permute the characters by using writing plaintext in one way and reading it in another way The permutation is done on the whole plaintext to create the whole ciphertext. Another method is to divide the plaintext into groups of predetermined size, called blocks, and then use a key to permute the characters in each block separately.

Example 3.25

Alice needs to send the message "Enemy attacks tonight" to Bob..

e	n	e	m	y		a	t	t	a	c		k	s	t	o	n		i	g	h	t	z

The key used for encryption and decryption is a permutation key, which shows how the character are permuted.

Encryption ↓

3	1	4	5	2
1	2	3	4	5

↑ Decryption

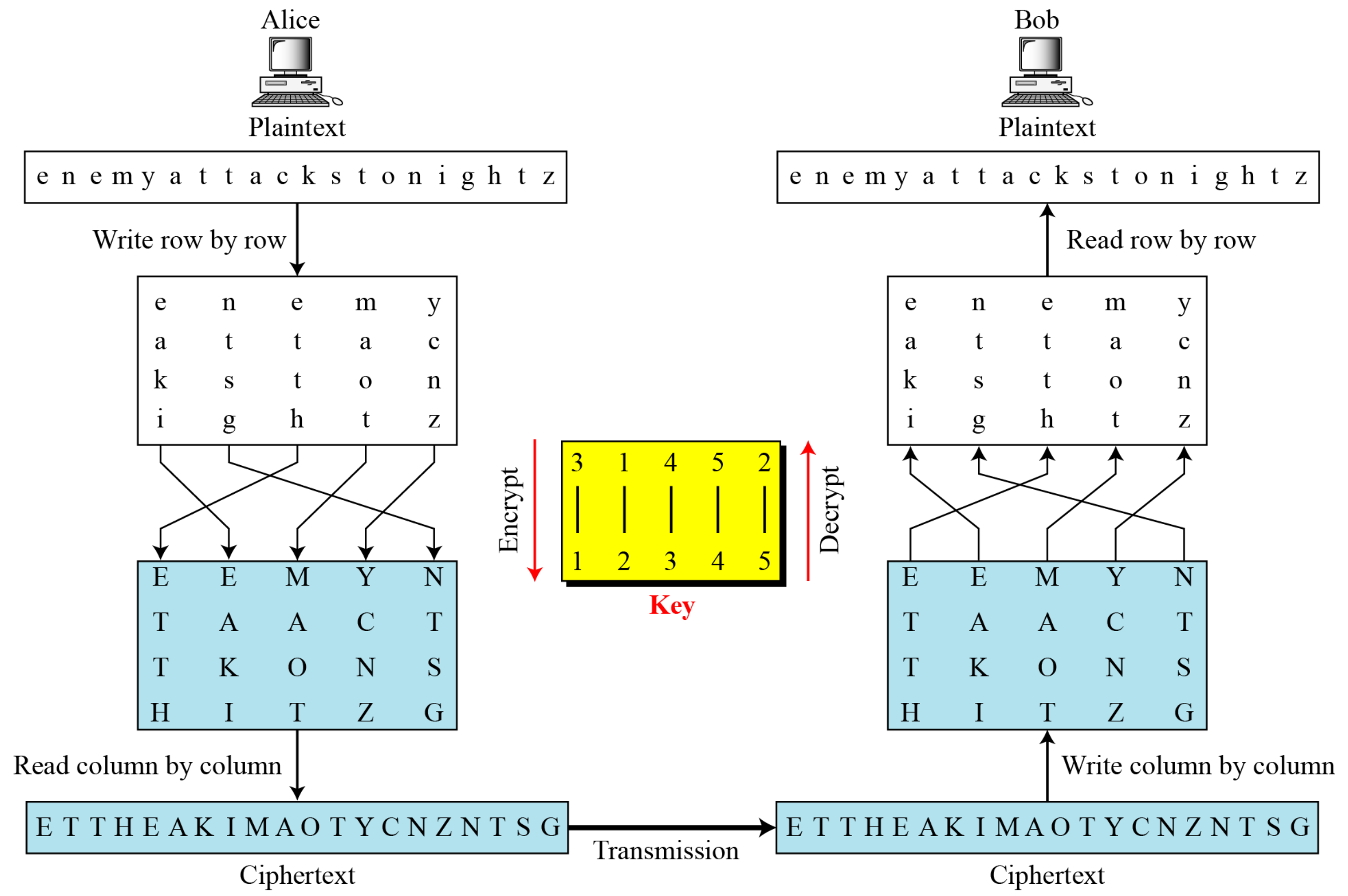
The permutation yields

E	**E**	**M**	**Y**	**N**		**T**	**A**	**A**	**C**	**T**		**T**	**K**	**O**	**N**	**S**		**H**	**I**	**T**	**Z**	**G**

3.3.3 Combining Two Approaches

Example 3.26

Figure 3.21

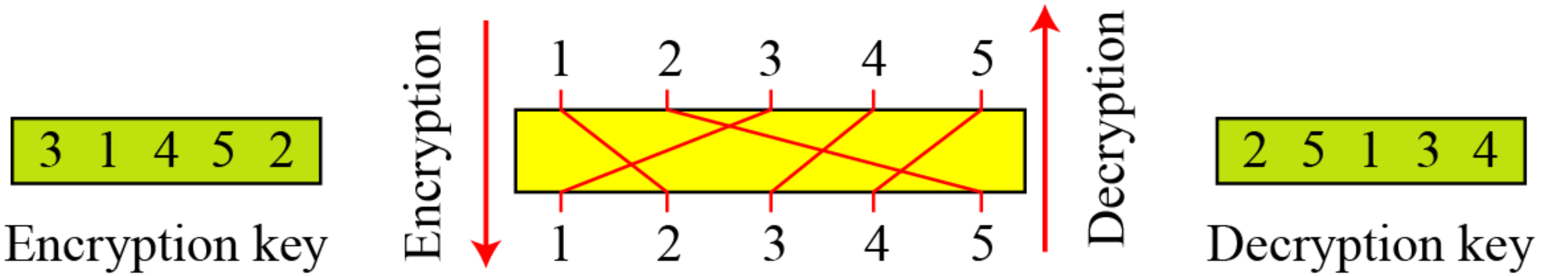


3.3.3 Continued

Keys

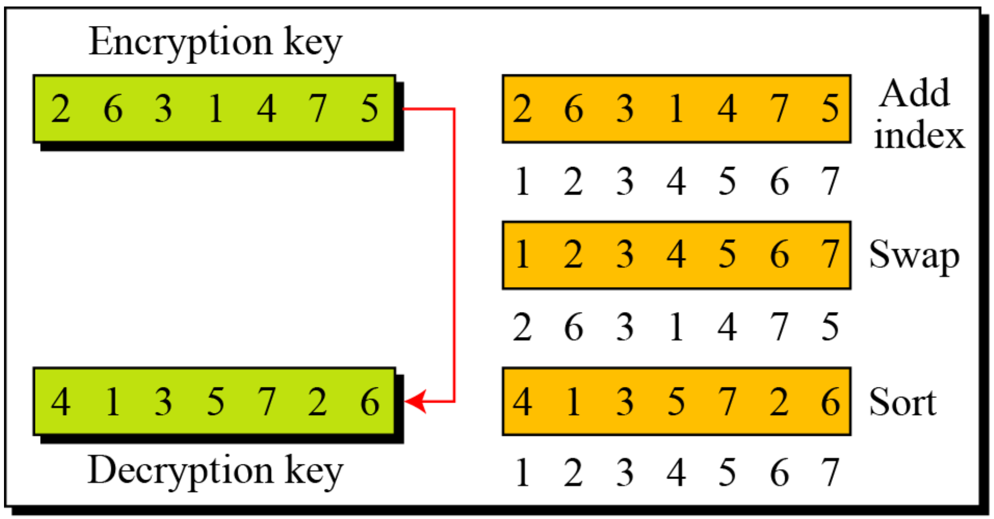
In Example 3.27, a single key was used in two directions for the column exchange: downward for encryption, upward for decryption. It is customary to create two keys.

Figure 3.22 ***Encryption/decryption keys in transpositional ciphers***



3.3.3 Continued

Figure 3.23 ***Key inversion in a transposition cipher***



a. Manual process

```
Given: EncKey [index]
index ← 1
while (index ≤ Column)
  {
    DecKey[EncKey[index]] ← index
    index ← index + 1
  }
Return : DecKey [index]
```

b. Algorithm

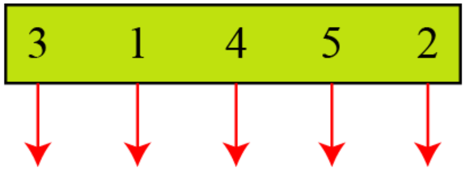
3.3.3 Continued

Using Matrices

We can use matrices to show the encryption/decryption process for a transposition cipher.

Example 3.27

Figure 3.24 ***Representation of the key as a matrix in the transposition cipher***



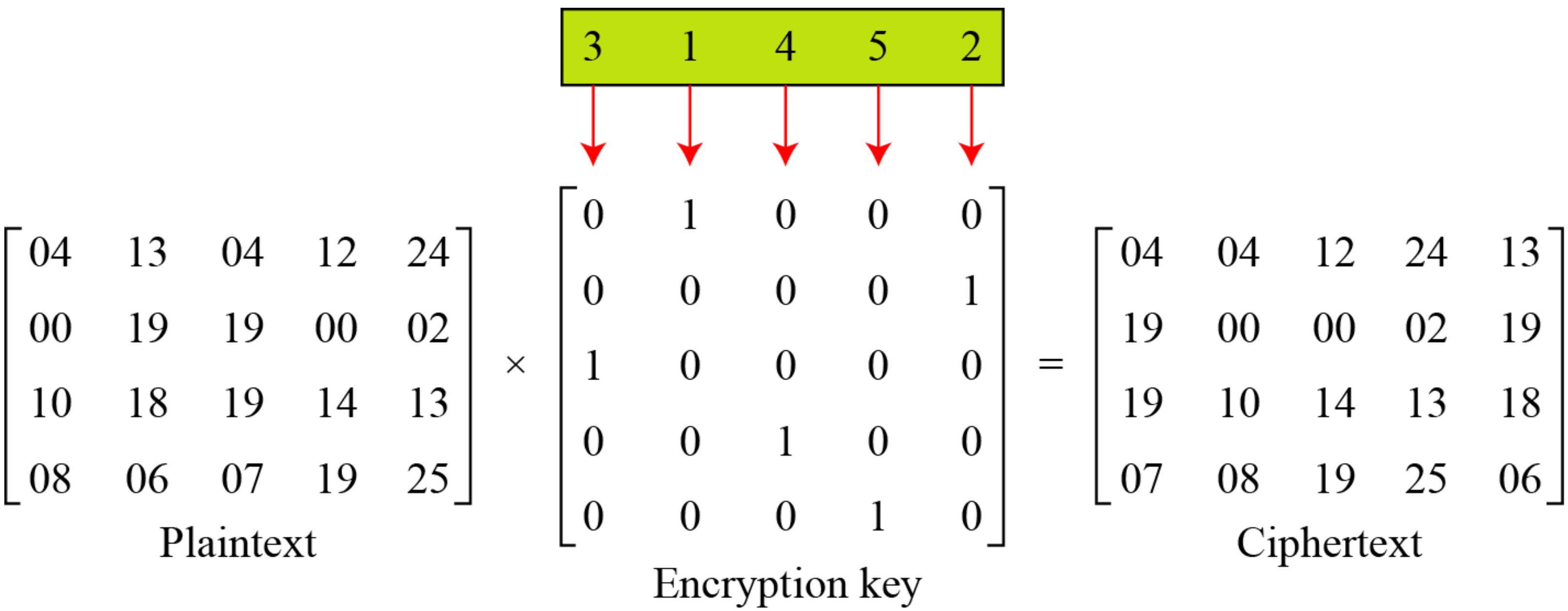
$$\underbrace{\begin{bmatrix} 04 & 13 & 04 & 12 & 24 \\ 00 & 19 & 19 & 00 & 02 \\ 10 & 18 & 19 & 14 & 13 \\ 08 & 06 & 07 & 19 & 25 \end{bmatrix}}_{\text{Plaintext}} \times \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{Encryption key}} = \underbrace{\begin{bmatrix} 04 & 04 & 12 & 24 & 13 \\ 19 & 00 & 00 & 02 & 19 \\ 19 & 10 & 14 & 13 & 18 \\ 07 & 08 & 19 & 25 & 06 \end{bmatrix}}_{\text{Ciphertext}}$$

3.3.3 *Continued*

Example 3.27

Figure 3.24 shows the encryption process. Multiplying the 4 × 5 plaintext matrix by the 5 × 5 encryption key gives the 4 × 5 ciphertext matrix.

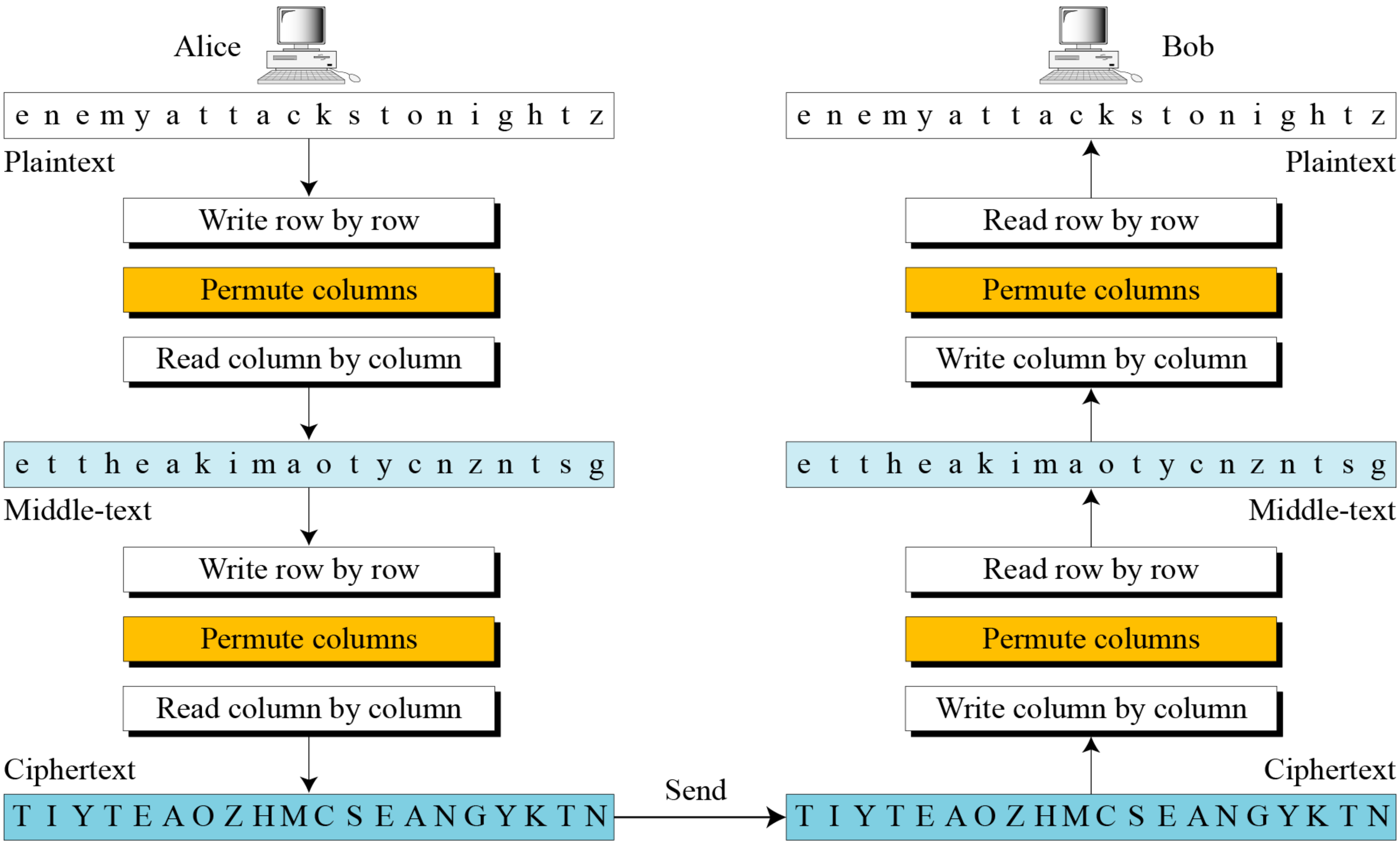
Figure 3.24 ***Representation of the key as a matrix in the transposition cipher***



3.3.3 Continued

Double Transposition Ciphers

Figure 3.25 *Double transposition cipher*



3-4 STREAM AND BLOCK CIPHERS

The literature divides the symmetric ciphers into two broad categories: stream ciphers and block ciphers. Although the definitions are normally applied to modern ciphers, this categorization also applies to traditional ciphers.

Topics discussed in this section:

3.4.1 Stream Ciphers
3.4.2 Block Ciphers
3.4.3 Combination

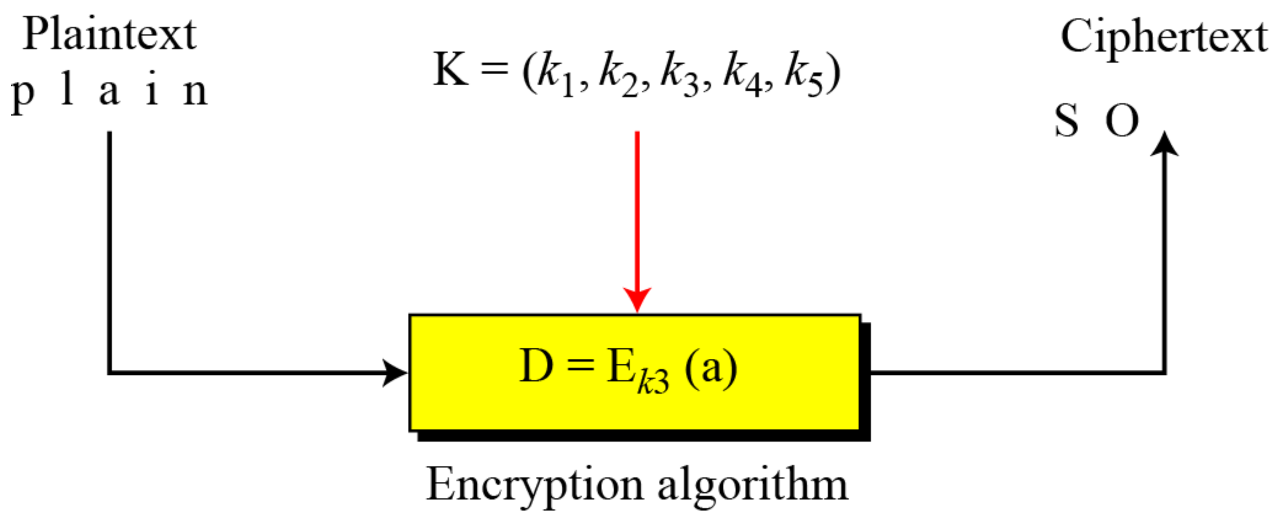
3.4.1 *Stream Ciphers*

Call the plaintext stream P, the ciphertext stream C, and the key stream K.

$$P = P_1P_2P_3, \ldots \qquad C = C_1C_2C_3, \ldots \qquad K = (k_1, k_2, k_3, \ldots)$$

$$C_1 = E_{k1}(P_1) \qquad C_2 = E_{k2}(P_2) \qquad C_3 = E_{k3}(P_3) \ \ldots$$

Figure 3.26 ***Stream cipher***



Example 3.30

Additive ciphers can be categorized as stream ciphers in which the key stream is the repeated value of the key. In other words, the key stream is considered as a predetermined stream of keys or K = (k, k, …, k). In this cipher, however, each character in the ciphertext depends only on the corresponding character in the plaintext, because the key stream is generated independently.

Example 3.31

The monoalphabetic substitution ciphers discussed in this chapter are also stream ciphers. However, each value of the key stream in this case is the mapping of the current plaintext character to the corresponding ciphertext character in the mapping table.

3.4.1 Continued

Example 3.32

Vigenere ciphers are also stream ciphers according to the definition. In this case, the key stream is a repetition of m values, where *m* is the size of the keyword. In other words,

$$K = (k_1, k_2, \ldots k_m, k_1, k_2, \ldots k_m, \ldots)$$

Example 3.33

We can establish a criterion to divide stream ciphers based on their key streams. We can say that a stream cipher is a monoalphabetic cipher if the value of k_i does not depend on the position of the plaintext character in the plaintext stream; otherwise, the cipher is polyalphabetic.

3.4.1 Continued

Example 3.33 (Continued)

- **Additive ciphers are definitely monoalphabetic because k_i in the key stream is fixed; it does not depend on the position of the character in the plaintext.**

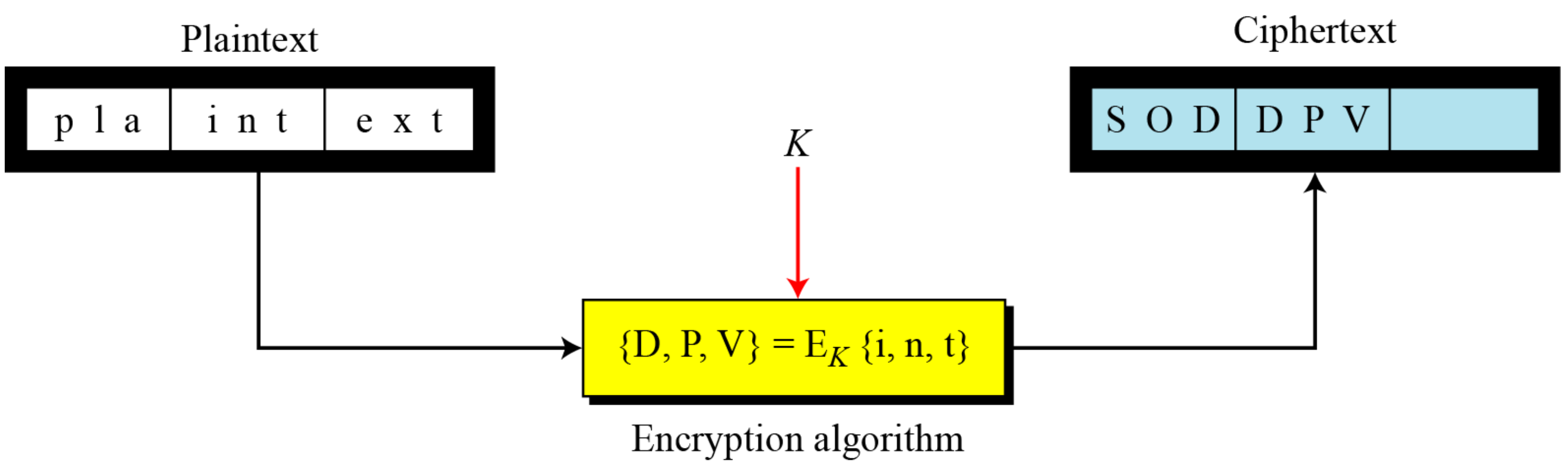
- **Monoalphabetic substitution ciphers are monoalphabetic because k_i does not depend on the position of the corresponding character in the plaintext stream; it depends only on the value of the plaintext character.**

- **Vigenere ciphers are polyalphabetic ciphers because k_i definitely depends on the position of the plaintext character. However, the dependency is cyclic. The key is the same for two characters *m* positions apart.**

3.4.2 Stream Ciphers

In a block cipher, a group of plaintext symbols of size *m* (*m* > 1) are encrypted together creating a group of ciphertext of the same size. A single key is used to encrypt the whole block even if the key is made of multiple values. Figure 3.27 shows the concept of a block cipher.

Figure 3.27 ***Block cipher***



Example 3.34

Playfair ciphers are block ciphers. The size of the block is $m = 2$. Two characters are encrypted together.

Example 3.35

Hill ciphers are block ciphers. A block of plaintext, of size 2 or more is encrypted together using a single key (a matrix). In these ciphers, the value of each character in the ciphertext depends on all the values of the characters in the plaintext. Although the key is made of $m \times m$ values, it is considered as a single key.

Example 3.36

From the definition of the block cipher, it is clear that every block cipher is a polyalphabetic cipher because each character in a ciphertext block depends on all characters in the plaintext block.

3.4.3 Combination

In practice, blocks of plaintext are encrypted individually, but they use a stream of keys to encrypt the whole message block by block. In other words, the cipher is a block cipher when looking at the individual blocks, but it is a stream cipher when looking at the whole message considering each block as a single unit.