

CS 456 : Advanced Algorithms

Problem Solving Session #00

Assigned Date : Tuesday, August 21, 2017
Due Date : Tuesday, September 05, 2017 @ 09:29:59 a.m.
 A hard copy submission at the beginning of class.

Take Home

- Q1. [25 points] Plot the functions $\lg(n)$ and $n^{0.49}$ on a linear graph for $1 \leq n \leq 25$, and comment on the relative growth of the two functions.
- Q2. [25 points] Plot the functions $\log(n^2)$ and $\log^2 n$ on a semi-log graph for $1 \leq n \leq 10,000$, and comment on the relative growth of the two functions.
- Q3. [10 points] Assume a computer that can perform 10^{10} operations per second. Find the largest input size n such that the result can be computed on this machine within an hour using each of the following five algorithms.
- $T_1(n) = n^2$
 - $T_2(n) = \sqrt{n}$
 - $T_3(n) = n \lg n$
 - $T_4(n) = 2^n$
 - $T_5(n) = 2^{2^n}$
- Q4. [15 points] Prove $\sum_{t=1}^n \frac{1}{t^2} \leq 2 - \frac{1}{n}$ using weak induction.
- Q5. [15 points] Prove $\sqrt{2}$ is irrational using proof by contradiction.
 (hint: Assume $\sqrt{2} = \frac{m}{n}$, where $\gcd(m, n) = 1; m, n \in \mathbb{Z}$)
- Q6. [15 points] Prove $2^x \geq x^2$ for $x \geq 4$ using induction.
- Q7. [10 points] Let $f(n), g(n)$, and $h(n)$ are asymptotically positive functions. Prove if $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$ then $f(n) = \Theta(h(n))$. (hint: Use the formal definition of Θ)
- Q8. [10 points] Let $f(n)$ and $g(n)$ are asymptotically positive functions. Prove $f(n) = \Theta(g(n))$ iff $g(n) = \Theta(f(n))$.
- Q9. [15 points] Using direct proof, prove that for any two integers $a, b \in \mathbb{Z}$, if both a , and b are **odd**, then the product ab is also odd. (hint: A odd number $y = 2x + 1; \exists x \in \mathbb{Z}$).
- Q10. [20 points] Prove the following properties of asymptotic growth. (hint: Use the formal definitions)
- [5 points] If $f(n) \in O(g(n))$ and $g(n) \in O(h(n))$, then $f(n) \in O(h(n))$.
 - [5 points] If $f(n) \in \Omega(g(n))$ and $g(n) \in \Omega(h(n))$, then $f(n) \in \Omega(h(n))$.
 - [5 points] If $f(n) \in \Theta(g(n))$ and $g(n) \in \Theta(h(n))$, then $f(n) \in \Theta(h(n))$.
 - [5 points] If $f(n) \in O(h(n))$ and $g(n) \in O(h(n))$, then $f(n) + g(n) \in O(h(n))$.