CS 456: Advanced Algorithms

Problem Solving Session #01

Total Points: 160

Take Home

Q1. [**25 points**] Plot the functions $lg(n)$ and $n^{0.49}$ on a linear graph for $1 \leq n \leq 25$, and comment on the relative growth of the two functions.

Q2. [**25 points**] Plot the functions $lg(n)$ and $n^{0.33}$ on a semi-log graph for $1 \leq n \leq 10,000$, and comment on the relative grown of the two functions.

Q3. [**10 points**] Assume a computer that can perform 10^{10} operations per second. Find the largest input size n such that the result can be computed on this machine within an hour using each of the following five algorithms.

- $T_1(n) = n^2$
- $T_2(n) = \sqrt{n}$
- $T_3(n) = nlgn$
- $T_4(n) = 2^n$
- $T_5(n) = 2^{2^n}$

Q4. [**15 points**] Prove $\sum_{t=1}^{n} \frac{1}{t^2} \leq 2 - \frac{1}{n}$ using weak induction.

Q5. [**15 points**] Prove $\sqrt{2}$ is irrational using proof by contradiction.
(*hint: Assume* $\sqrt{2} = \frac{m}{n}$, *where* $gcd(m,n) = 1; m, n \in \mathbb{Z}$)

Q6. [**15 points**] Prove $2^x \geq x^2$ for $x \geq 4$ using induction.

Q7. [**10 points**] Let $f(n), g(n),$ and $h(n)$ are asymptotically positive functions. Prove if $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$ then $f(n = \Theta(h(n)))$. (*hint: Use the formal definition of* Θ)

Q8. [**10 points**] Let $f(n)$ and $g(n)$ are asymptotically positive functions. Prove $f(n) = \Theta(g(n))$ **iff** $g(n) = \Theta(f(n))$.

Q9. [**15 points**] Using direct proof, prove that for any two integers $a, b \in \mathbb{Z}$, if both a, and b are **odd**, them the product ab is also odd. (*hint: A odd number* $y = 2x + 1; \exists x \in \mathbb{Z}$).

Q10. [**20 points**] Prove the following properties of asymptotic growth. (*hint: Use the formal definitions*)

- [**5 points**] If $f(n) \in O(g(n))$ and $g(n) \in O(h(n))$, then $f(n) \in O(h(n))$.
- [**5 points**] If $f(n) \in \Omega(g(n))$ and $g(n) \in \Omega(h(n))$, then $f(n) \in \Omega(h(n))$.
- [**5 points**] If $f(n) \in \Theta(g(n))$ and $g(n) \in \Theta(h(n))$, then $f(n) \in \Theta(h(n))$.
- [**5 points**] If $f(n) \in O(h(n))$ and $g(n) \in O(h(n))$, then $f(n) + g(n) \in O(h(n))$.