CS 456: Advanced Algorithms

Homework #01

Assigned Date : Friday, September 05, 2014
Due Date : Tuesday, September 16, 2014 @ 12:29:59 p.m.

Instructions

- Do your own work.
- It is highly recommended to produce your answers using a word processor as much as possible. Handwritten parts, if present, should be clear both in terms of readability as well as general appearance.
- You may email your solutions to the instructor or turn-in a printed, stapled copy by **Tuesday, September 16, 2014 @ 12:29:59 p.m.** Include your name and the last 3 digits of your SIUE ID in the solutions sheet.
- Answer all questions.
- Total points: [**270 points**]

Questions

Q1. For each of the following cases, show whether $f = O(g)$ or $f = \Omega(g)$ or $f = \Theta(g)$. [**80 points**]

	$f(n)$	$g(n)$
(a)	$10log(n)$	$log(n^2)$
(b)	$n^{1/2}$	$n^{2/3}$
(c)	$\sqrt{n}$	$(log(n))^3$
(d)	$n2^n$	3^n
(e)	$100n + log(n)$	$n + log(n)^2$
(f)	$n!$	2^n
(g)	$n^{0.1}$	$(log(n))^{10}$
(h)	$\sum_{i=1}^{n} i^k$	n^{k+1}

Q2. Consider the Fibonacci numbers $F_0, F_1, F_2, \ldots$ defined as,

$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$$

a. Use induction to prove that $F_n \geq 2^{0.5n}$ for $n \geq 6$ [**20 points**]
b. Find a constant $c < 1$ such that $F_n \leq 2^{cn}$ for all $n \geq 0$. Show your work. [**20 points**]
c. What is the largest c you can find for which $F_n = \Omega(2^{cn})$? [**15 points**]

Q3. Prove the equality $gcd(m, n) = gcd(n, m\ mod\ n)$ for every pair of positive integers m and n [**20 points**]

Q4. A peasant finds himself on a riverbank with a wolf, a goat, and a head of cabbage. He needs to transport all three to the other side of the river in his boat. However, the boat has room only for the peasant himself and one other item (either the wolf, the goat, or the cabbage). In his absence, the wolf would eat the goat, and the goat would eat the cabbage. Solve this problem for the peasant or prove it has no solution (A peasant is a vegetarian but does not like cabbage and hence can eat neither the goat nor the cabbage to help him solve the problem.) [**20 points**]

Q5. Write a pseudocode for an algorithm for finding real roots of equation $ax^2 + bx + c = 0$ for arbitrary real coefficients a, b, and c. Assume the existence of a square root function $sqrt(x)$ [**25 points**].

Q6. For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions. [**50 points**]

a. $(n^2 + 1)^{10}$
b. $2n\, lg(n+2)^2 + (n+2)^2 lg(\frac{n}{2})$
c. $\sqrt{10n^2 + 7n + 3}$
d. $2^{n+1} + 3^{n-1}$
e. $\lfloor log_2 n \rfloor$

Q7. List the following function according to their order of growth from the lowest to the highest: [**20 points**]

$$(n-2)!,\ \ 5lg(n+100)^{10},\ \ 2^{2n},\ \ 0.0001n^4 + 3n^3 + 1,\ \ ln^2 n,\ \ \sqrt[3]{n},\ \ 3^n$$