1. Review the relevant sections from Chapters 1.1 – 1.7, 2.1 – 2.5, 3, 4.1 – 4.4, and 5.1 in our text. Also review quizzes and written assignments.

2. Construct a truth table for \( p \oplus q \rightarrow q \).

3. Construct a truth table for \((\neg p \rightarrow q) \oplus q\).

4. Construct a truth table for \( p \rightarrow (q \land \neg r)\).

5. Sort the following in order by \( \Theta \), (smallest to largest).

\[ n \lg n, \ n^5, \ 2^{4 \lg n}, \ 2^n, \ 3n^3 + n^5, \ \lg n^5, \ n^{0.9} \]

6. Explain what it means for a set of operators, e.g., \( \land \), \( \lor \), and \( \neg \) to be a functionally complete set of propositional operators.

7. Convert the following proposition into conjunctive normal form: \((p \lor q) \rightarrow (r \land \neg s)\).

8. Using a truth table, show that the two propositions below are equivalent.

\[(a \land b) \lor (a \land c) \quad ((a \rightarrow b) \lor (a \rightarrow c)) \land a\]

9. What rule of inference is used in the following. If there are more than 100 CS majors, a course in advanced algorithms will be offered during the spring semester. If advanced algorithms is offered, at least one student will go to the chair and complain that CS is too hard. There are more than 100 CS majors at SIUE, therefore at least one student will go to the chair and complain that CS is too hard. (See the board for a list of rules.)

10. You meet two people, Helen and Harold. One of them always lies and the other always tells the truth. If you ask Helen, “does Harold always lie?” Explain why she will always say “yes”.

11. Be able to do Exercise 23 on page 54.


13. Below is the definition of Big-O.

\[ T(n) = O(g(n)) \text{ when } (\exists c > 0, n_0 \geq 0)(\forall n > n_0)(T(n) \leq cg(n)) \]

The following is a predicate that is a modification of the Big-O definition, called Big-L, with the \( \exists \) and \( \forall \) reversed.

\[ T(n) = L(g(n)) \text{ when } (\forall n > 0)(\exists c > 0)(T(n) \leq cg(n)) \]

Explain why \( T(n) = L(g(n)) \) as long as \( g(n) \) is any function whose range is the positive reals. Use \( T(n) = n^3 \) and \( g(n) = n^2 + 1 \) to illustrate that \( n^3 = L(n^2 + 1) \) with different values of \( n \). Would \( g(n) = n^2 \) work? Is Big-L a reasonable alternative to Big-O?

14. Consider the set \( A = \{(x, y) \mid (y = \lfloor \lg(x) \rfloor \land (0 < x < 32)) \} \), where the universe is the set of natural numbers. Then give the smallest sets \( B \) and \( C \) such that \( A \subseteq B \times C \).

15. Is the following a proof by contradiction or a counter example?

**Hypothesis:** The sum of two positive, even integers is even.

**Proof:** Let \( A \) and \( B \) be positive even numbers, and assume that \( A + B \) is odd. Since \( A \) and \( B \) are even numbers, there are integers \( x \) and \( y \) such that \( A = 2x \) and \( B = 2y \). Then \( A + B = 2x + 2y = 2(x + y) \). But then \( A + B \) is not odd and the hypothesis must be true.
16. Which of the following sets are countable and which are uncountable?
(a) \{1, 2, 3\}, (b) \{x \in \mathbb{Z} | x > 0\}, (c) \mathcal{P}(\mathbb{Z}), (d) \mathbb{R}, (e) \{(x, y) | x \in \mathbb{Z} \land y \in \mathbb{Z}\}, (f) \mathbb{Z} \times \mathbb{R}

17. Be able to answer the following questions regarding functions: Exercises 1, 2 and 6 on Page 146, and 18 and 19 on page 147.

18. Be sure to know the meaning of the terms, bijection, one-to-one, on-to, and inverse function.

19. Given the code for insertion sort below, show the contents of array \(A\) after five complete passes through the outer for loop.

\[
\begin{array}{cccccccc}
\text{A} &=& 11 & 20 & 2 & 15 & 31 & 40 & 50 & 25 \\
\text{Index} &=& 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7
\end{array}
\]

```c
void insertion_sort(array data, int N)
int cnt1, cnt2;
element item;
for (cnt1 = 1; cnt1 < N; cnt1++) {
    cnt2 = cnt1 - 1;
    item = data[cnt1];
    while ( (cnt2 >= 0) && (item < data[cnt2]) ) {
        data[cnt2 + 1] = data[cnt2];
        cnt2--;
    } // end while
    data[cnt2 + 1] = item;
} // end for loop

20. Referring to the code for insertion sort above explain informally why the worst-case performance is \(\Theta(N^2)\) and the best case performance is \(\Theta(N)\). Do this by giving examples of arrays that give these results.

21. Show that \(n \log n = O(n^2)\) by finding a value for \(n_0\) and a constant \(C\).

22. Show that \(\log(x^3) = \Theta \log(x)\) by finding appropriate values for \(n_0, C_1,\) and \(C_2\).

23. Give the decimal value for each of these 8 bit twos-complement numbers. Remember some of them may be negative values.
(a) 1010 0001, (b) 0101 1111 (c) 1000 0010 (d) 0001 0110

24. Name four computer science applications for modular arithmetic.

25. Which of these pairs are relatively prime?
(a) 8 and 15 (b) 105 and 10 (c) 81 and 45 (d) 50 and 33

26. Find the multiplicative inverse of 8 modulus 13?

27. Given the function below defined on the natural numbers, construct a table showing the value of \(f(x)\) for \(x \in \{0, 1, 2, 3, 4, 5\}\).
\[
f(x) = \begin{cases} 
2 & \text{if } x = 0 \\
f(x-1) + 3x & \text{if } x > 0 
\end{cases}
\]

28. Prove by mathematical induction that \(4x^2 \geq 3x^2 + 2x + 2\) for \(x \geq 3\). Be sure to give the basis step, the inductive hypothesis, and the inductive step.